4 – Force Detection

ME-426 – Micro/Nanomechanical Devices

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Schedule

Week	Day	Theory			Question by
1	19.09	Introduction; Scaling laws, Fabrication*			
2	26.09	Transduction of the motion – Techniques			
3	03.10	Transduction of the motion – Techniques		Paper (1)	30.09.2022
4	10.10			Paper (1)	
5	17.10	Static deflection – Force, Linear behavior, Intro noise			
6	24.10	Station deflection – Stress		Paper (2)	21.10.2022
7	31.10	Dynamic behavior – Linear Analysis		Paper (3)	28.10.2022
8	08.11	Dynamic behavior – Stress influence on frequency			
9	14.11	Dynamic behavior – Mode coupling Dynamic behavior – Nonlinearities, Parametric drive			
10	21.11	Material properties – Size effects		Paper (4)	18.11.2022
11	28.11			Paper (5,6)	25.11.2022
12	05.12	Quality factor – Energy losses Thermomechanical noise, Detection schemes			
13	12.12			Paper (7)	09.12.2022
14	19.12			Paper (8, 9)	09.12.2022

Practical matters – Let's connect

- Smartphones
  - iPhone
  - Android
  - Blackberry

  Download and install the app "*PointSolutions*"

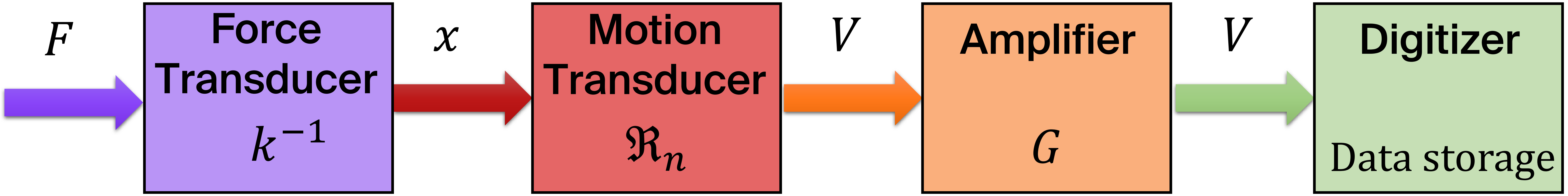
- Computers (or Windows Phones)
  - Go to responseware.eu
- Choose EUROPE Server (if asked)
- Join session – ME426
- When asked – do not sign in, do not input your name. Just enter as guest.

Introduction

- Force detection
- Transduction chain
- Linear behavior
- Lumped element model
- Nonlinear behavior
- Composite beams
- Accelerometers

Force detection

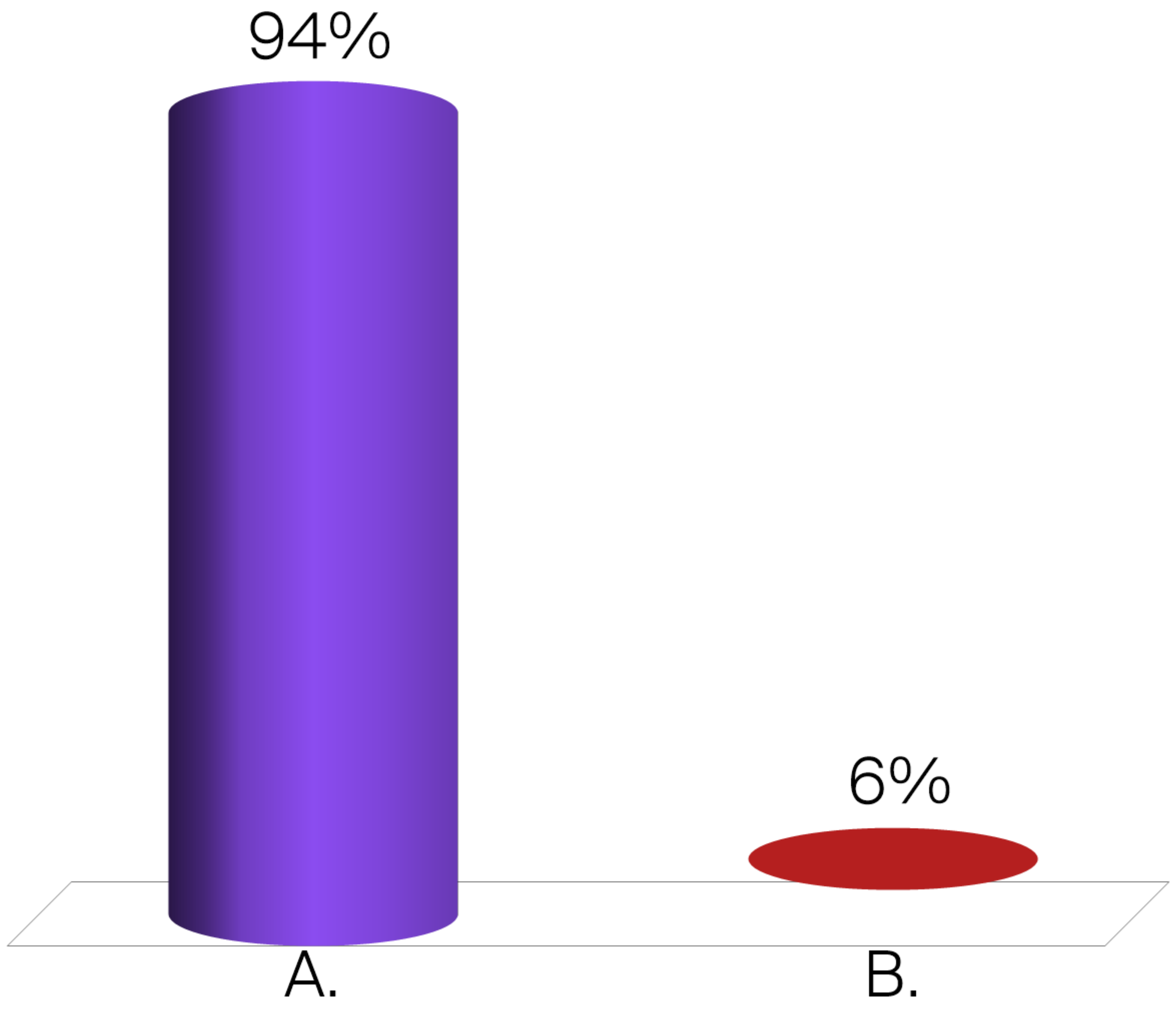
Force detection



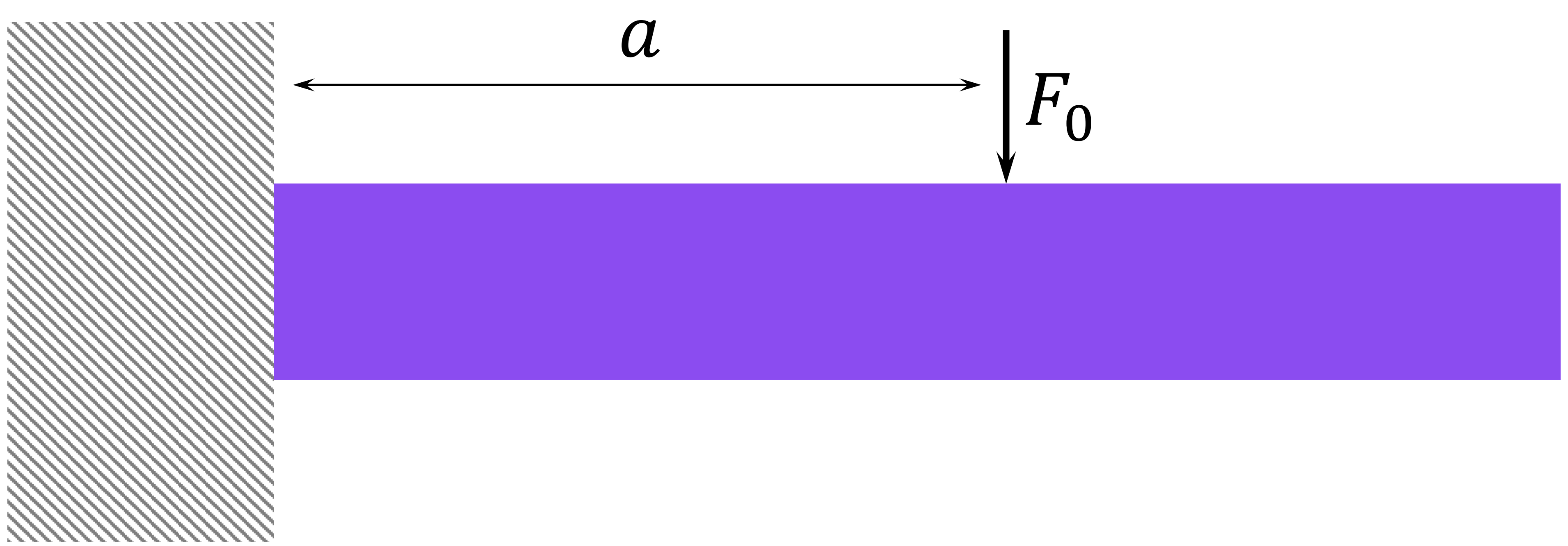
Which structure is better to detect a force?

A. Flexural beam

B. Elongational beam

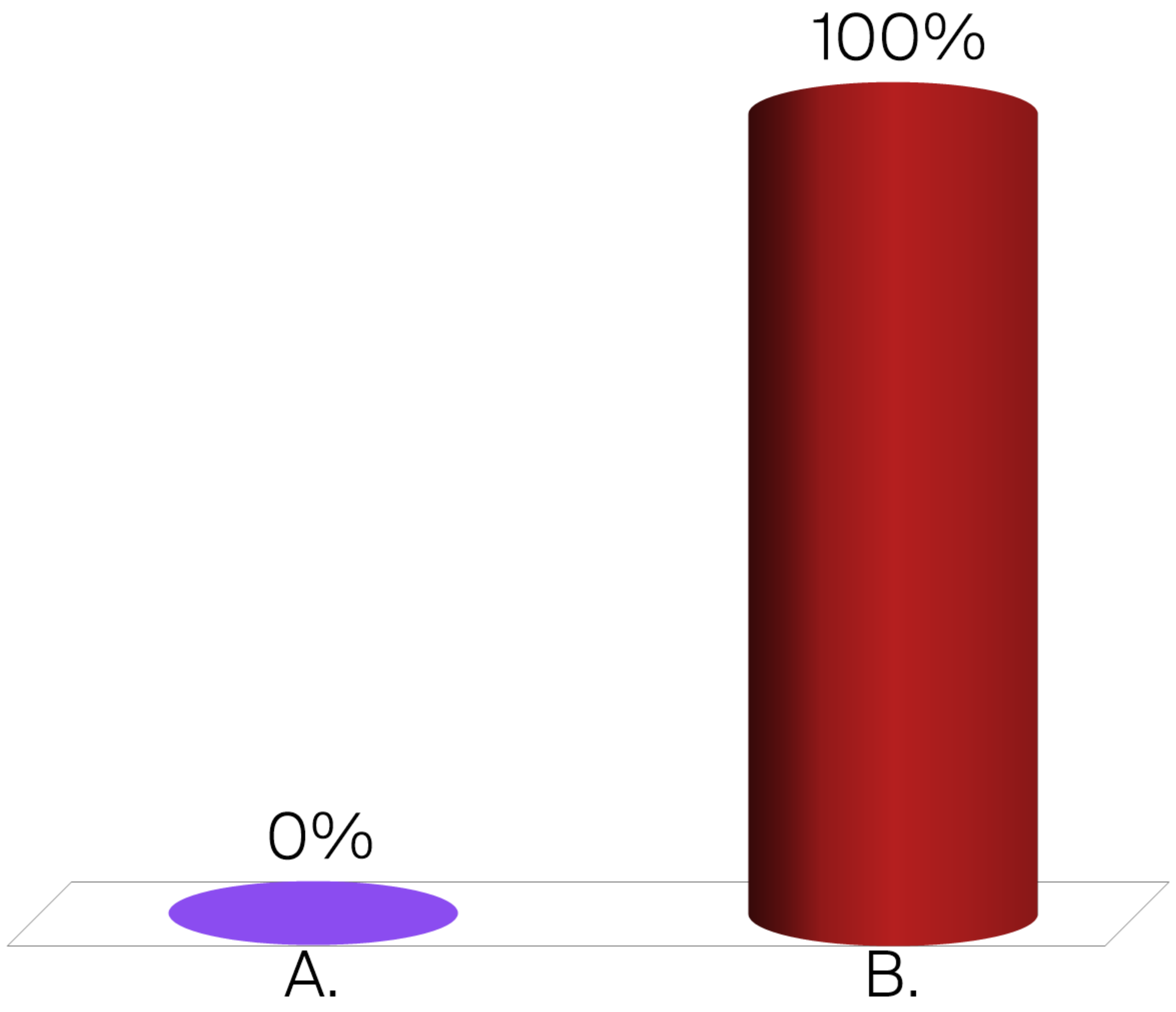


Is the minimum detectable force the same for all a?

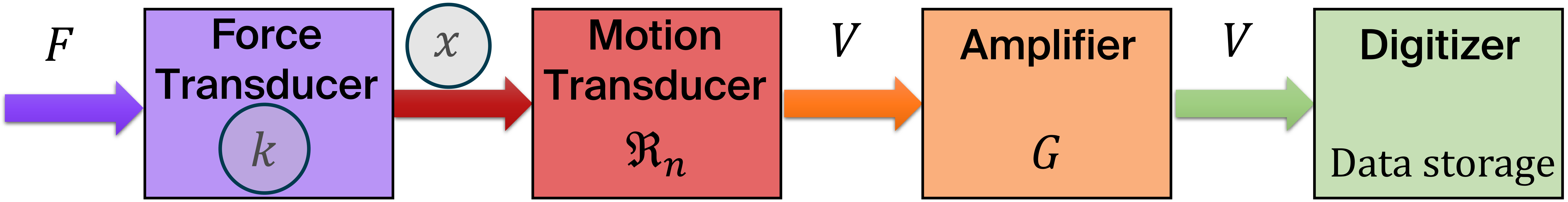


A. Yes

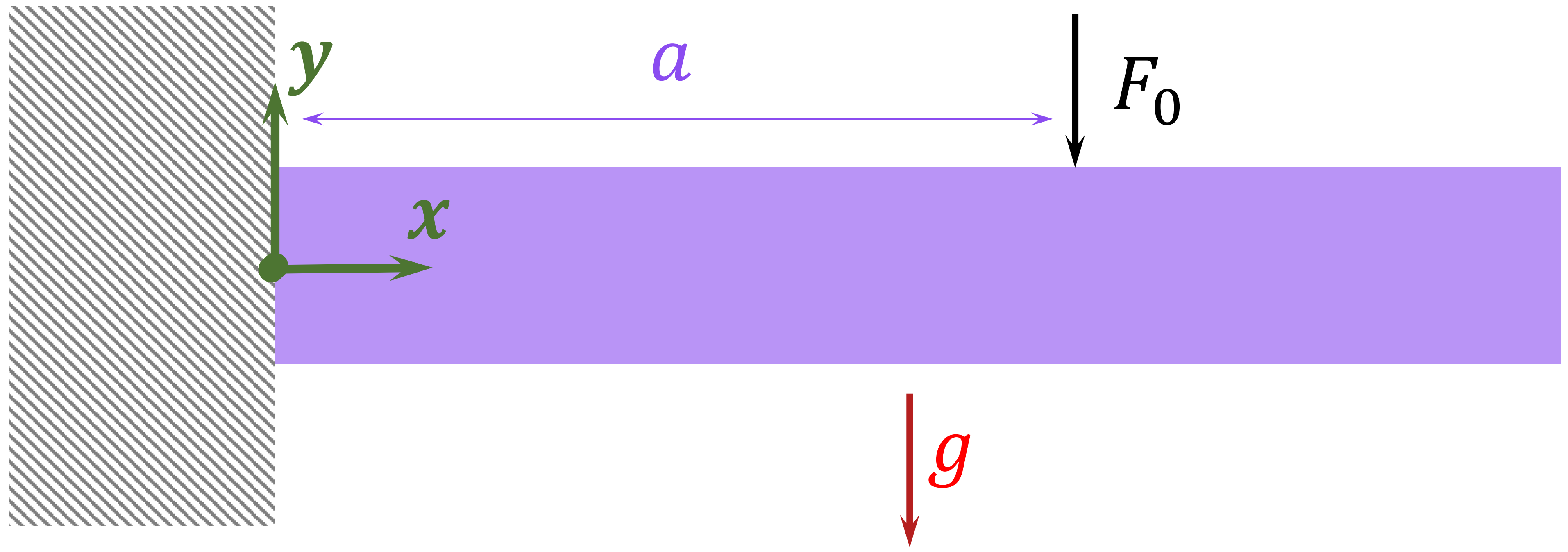
B. No



Force detection



Let’s do some basics



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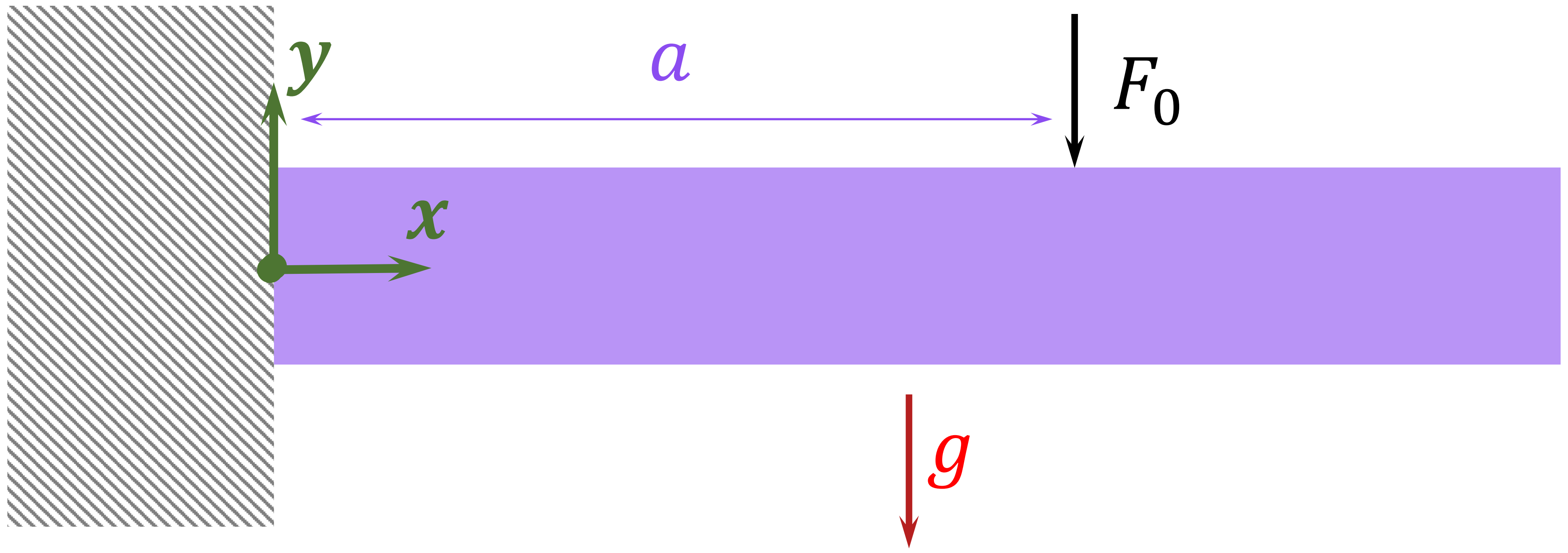

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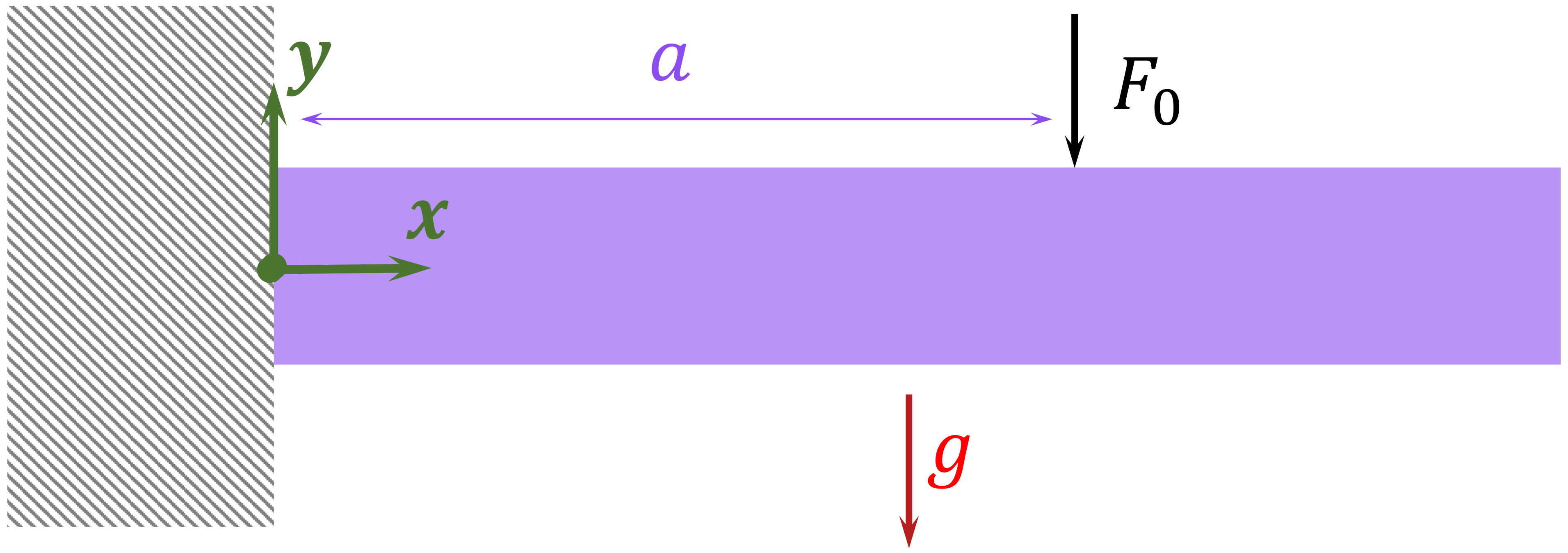

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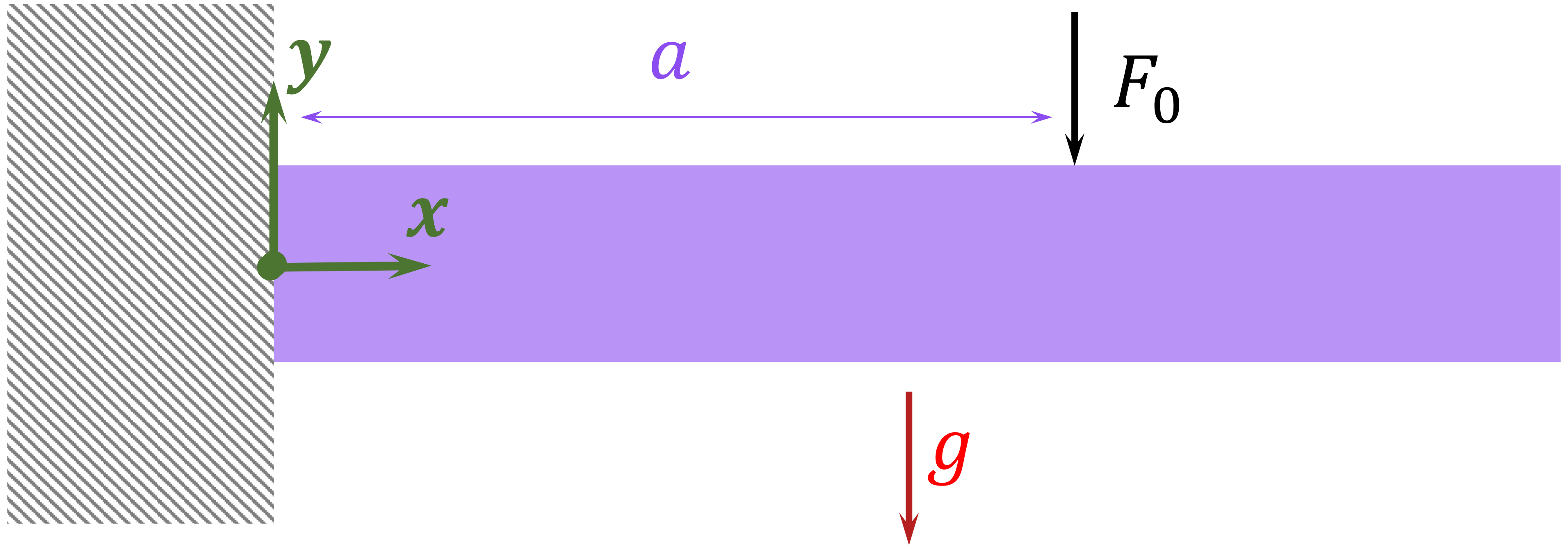
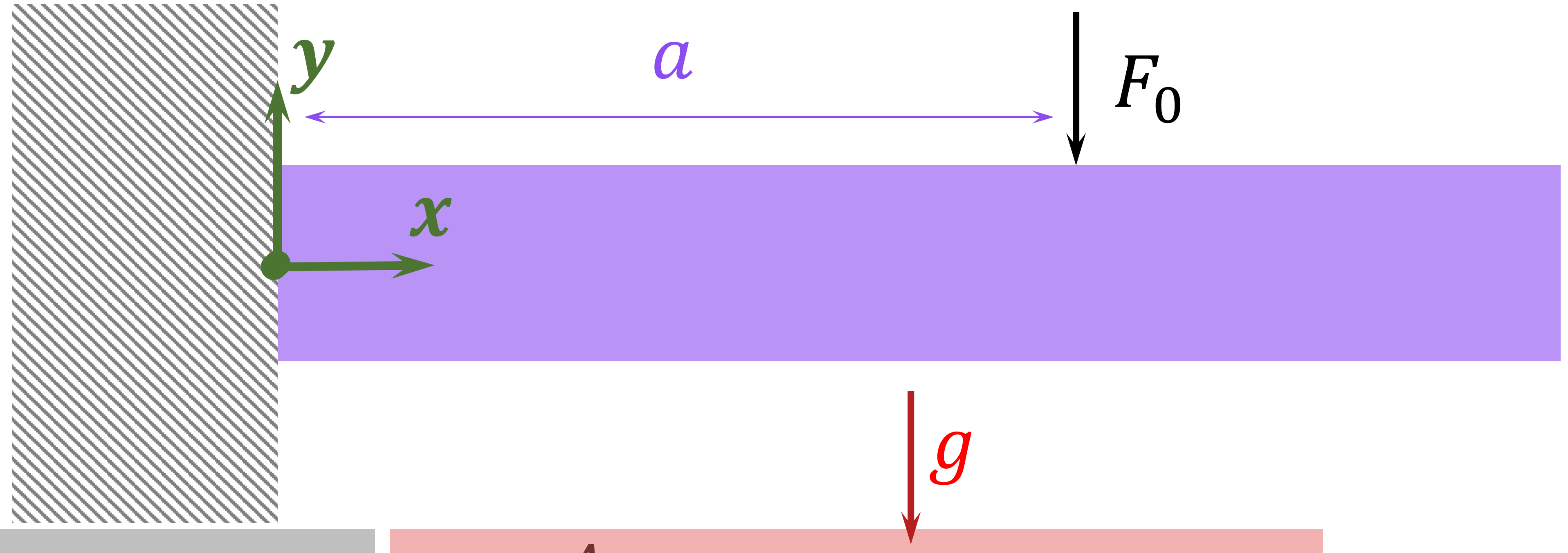


$$V(x) = \begin{cases} F_0 + \rho g A(L - x) & if\ x < a \\ \rho g A(L - x) & if\ x > a \end{cases}$$

$$M(x) = \begin{cases} F_0(x - a) - \frac{\rho g A}{2}(x - L)^2 & if\ x \leq a \\ -\frac{\rho g A}{2}(x - L)^2 & if\ x \geq a \end{cases}$$

$$w(x) = \begin{cases} \frac{F_0}{6EI}x^2(x - 3a) - \frac{\rho g A}{12EI}x^2(6L^2 - 4Lx + x^2) & if\ x \leq a \\ \frac{F_0}{6EI}a(-2a^2 - 3xa) - \frac{\rho g A}{12EI}x^2(6L^2 - 4Lx + x^2) & if\ x \geq a \end{cases}$$

Let's put some numbers



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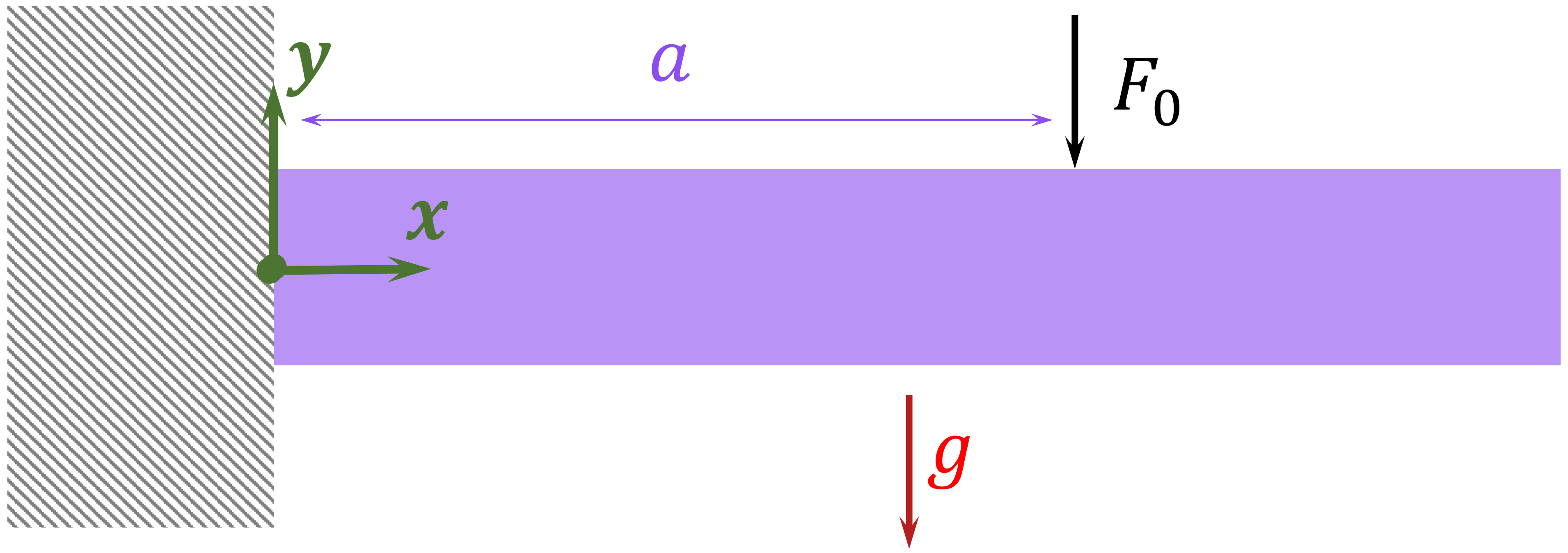
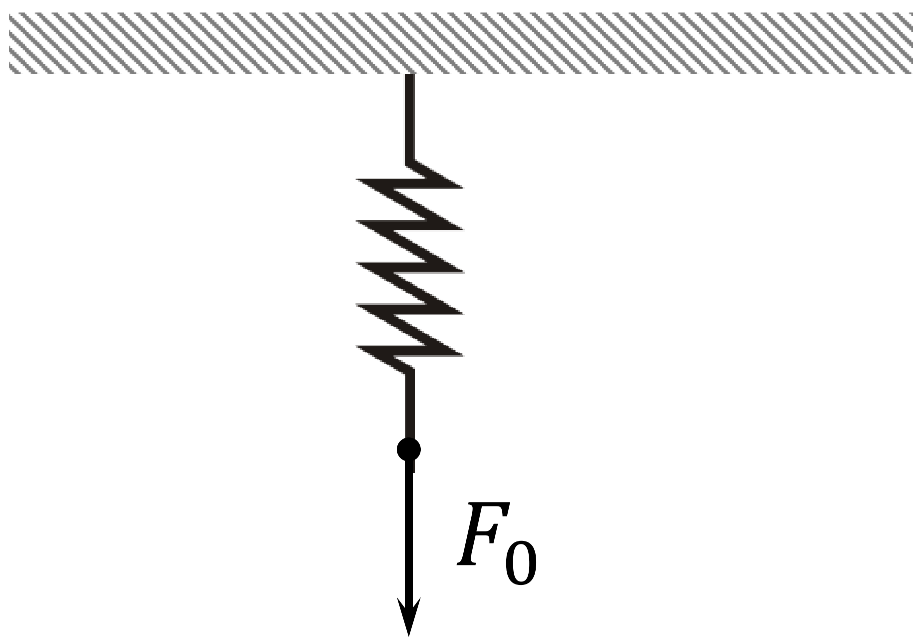
Device	Dimensions (m)	Max. deflection gravity (m)	Force to match gravity (N)
Diving board	$10 \times 1 \times 0.1$	0.3	$25 \cdot 10^3$
Tuning fork	$0.1 \times 10^{-3} \times 10^{-3}$	$3 \cdot 10^{-5}$	$2 \cdot 10^{-3}$
Microcantilever	$(500 \times 50 \times 5) \cdot 10^{-6}$	10^{-9}	$4 \cdot 10^{-9}$
Nanocantilever	$(10 \times 0.5 \times 0.1) \cdot 10^{-6}$	$3 \cdot 10^{-13}$	10^{-14}

In micro/nano scale, gravity and weight can be neglected – but maybe important for vibrations

Lump element model





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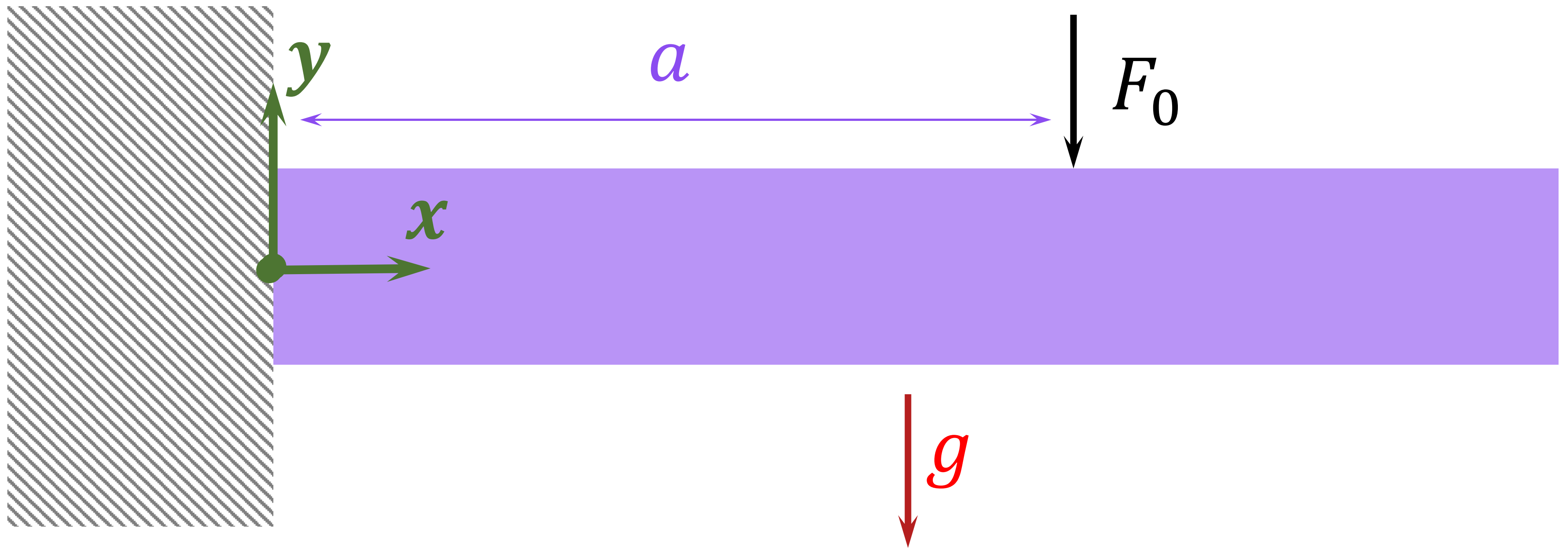

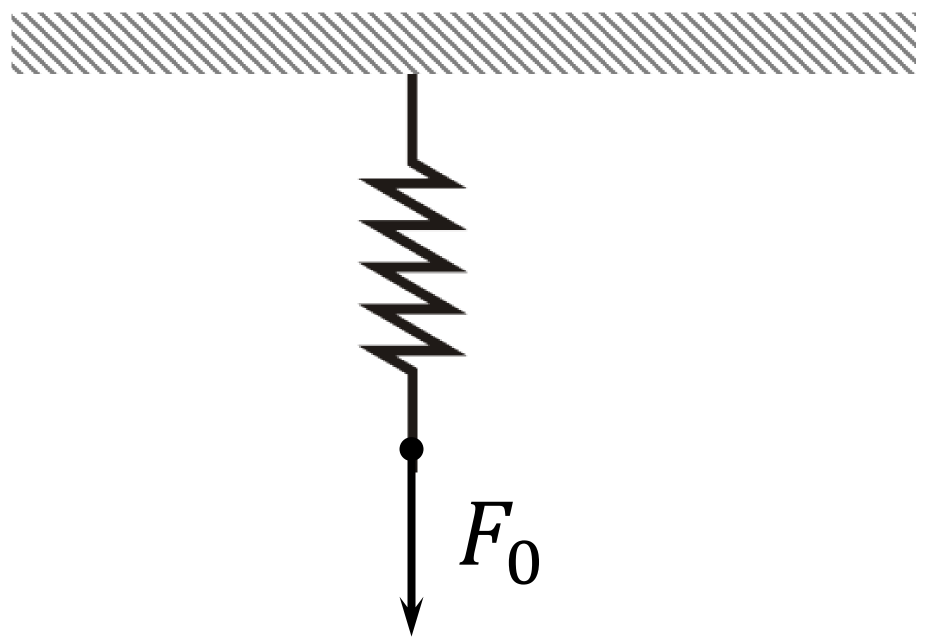

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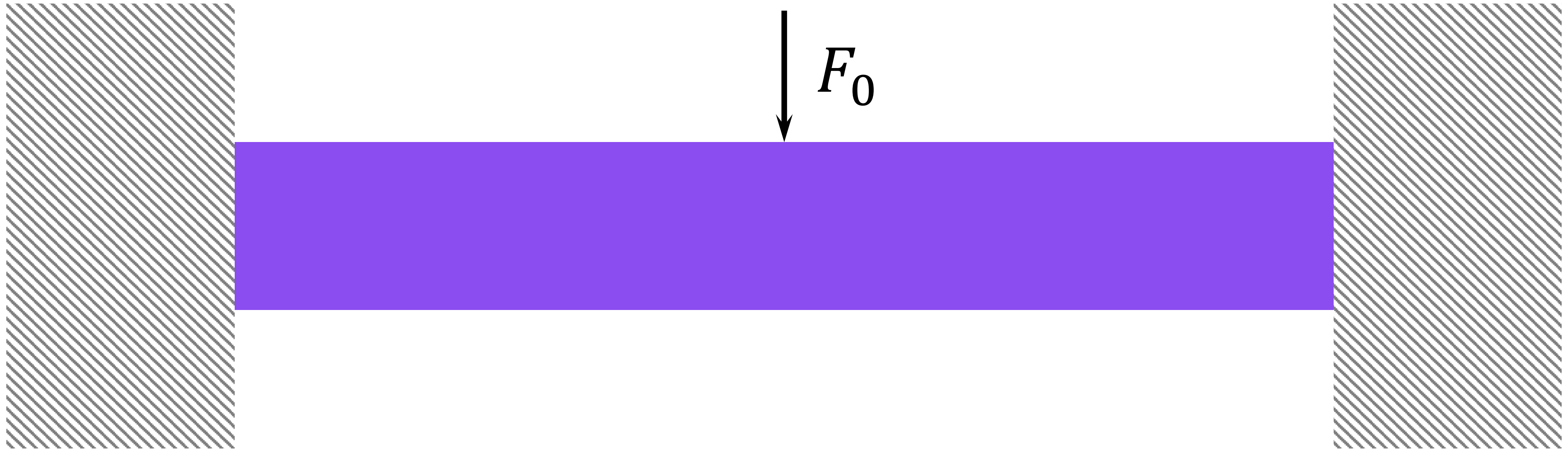
$$\boldsymbol{k = \frac{F_0}{w(x)}}$$

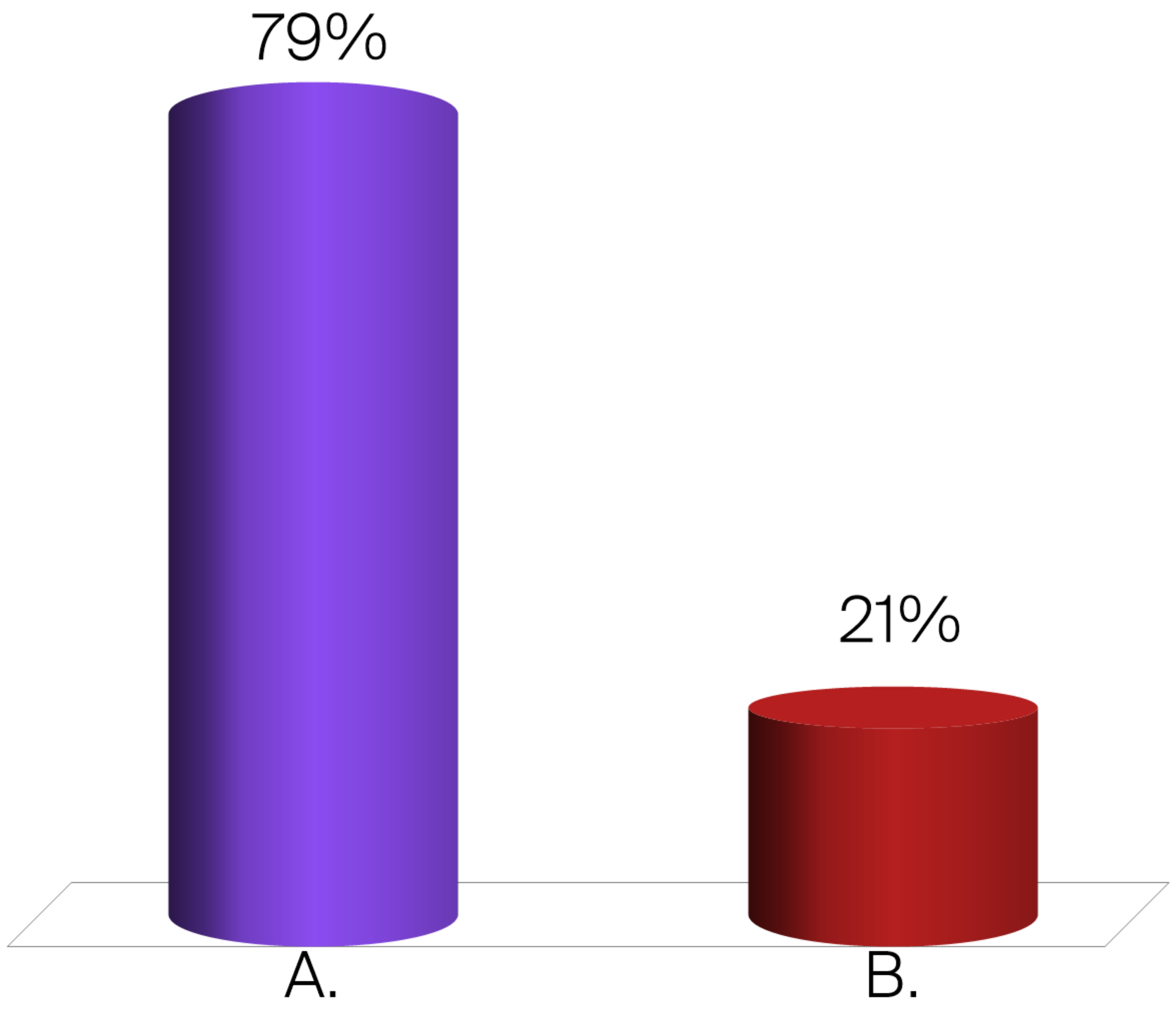
- Definition of elastic constant seems easy enough
- But what displacement should we take?
- Is the k different depending on where we apply the force?

Which structure is better to detect a force?

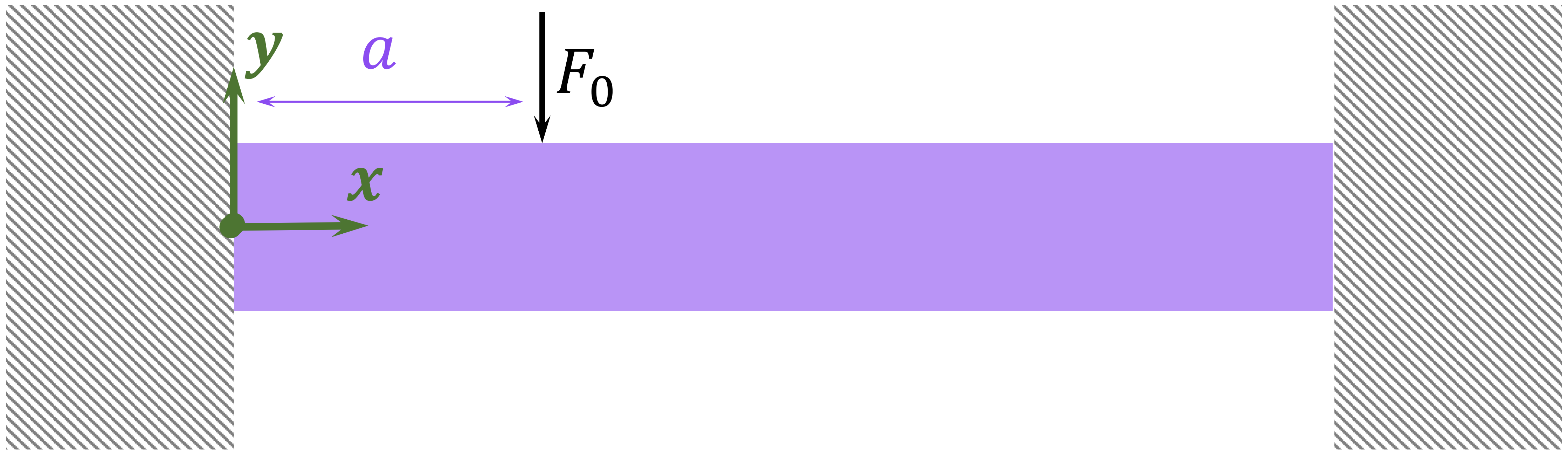
A. Cantilever beam

B. Clamped-Clamped beam





Clamped-Clamped beam



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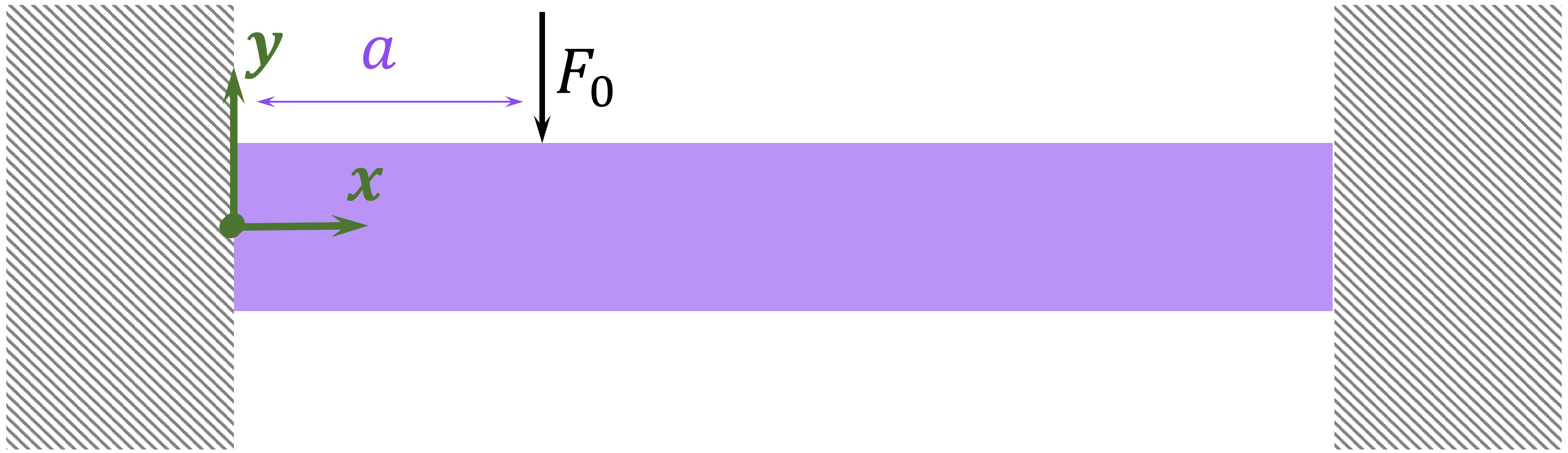

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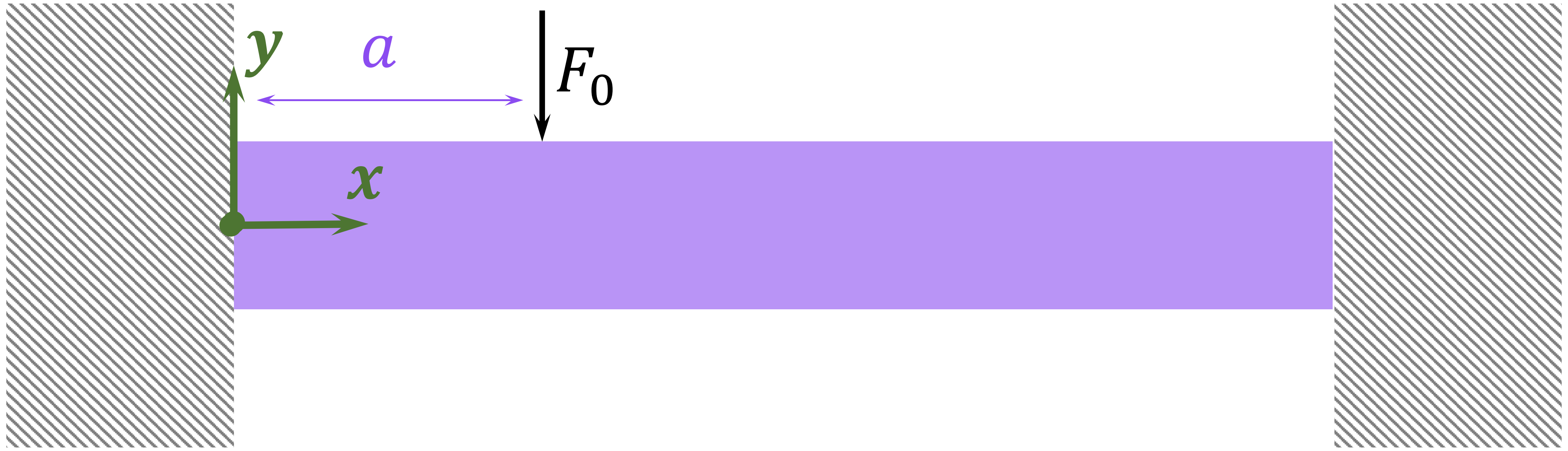

$$w(x) = \begin{cases} \frac{F_0}{6EI} x^2 \left(1 - \frac{a}{L}\right)^2 \left(-3a + x\left(1 + \frac{2a}{L}\right)\right) & if\ x \le a \\ \frac{F_0}{6EI} a^2 \left(1 - \frac{x}{L}\right)^2 \left(-3x + a\left(1 + \frac{2x}{L}\right)\right) & if\ x \ge a \end{cases}$$

- Definition of elastic constant seems easy enough

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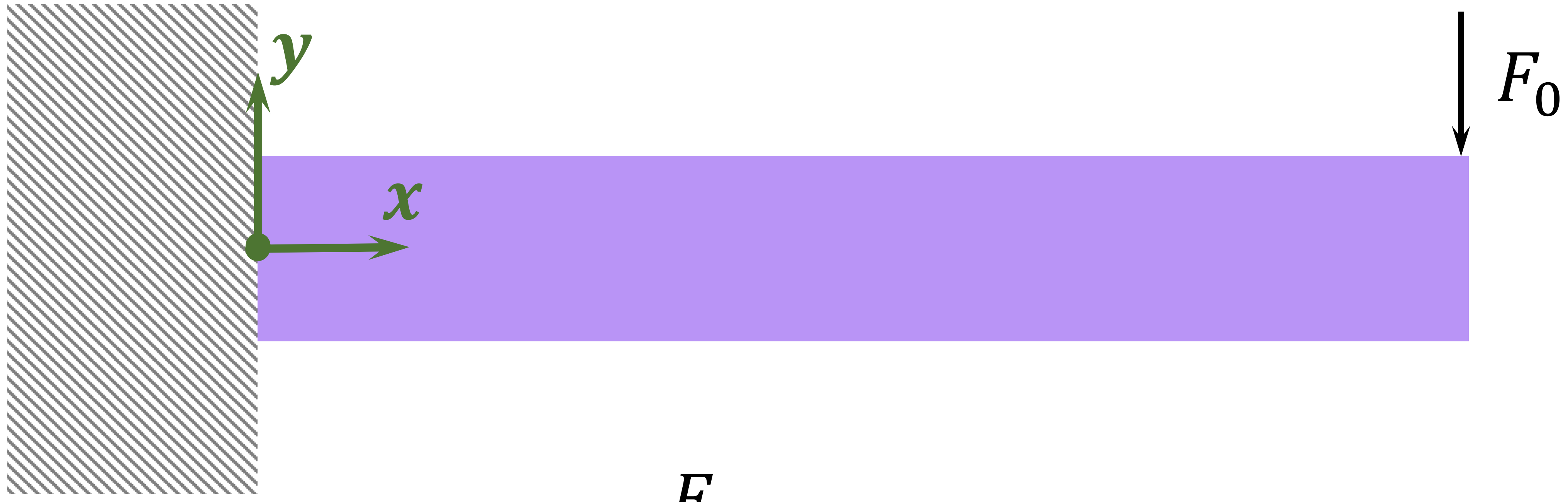
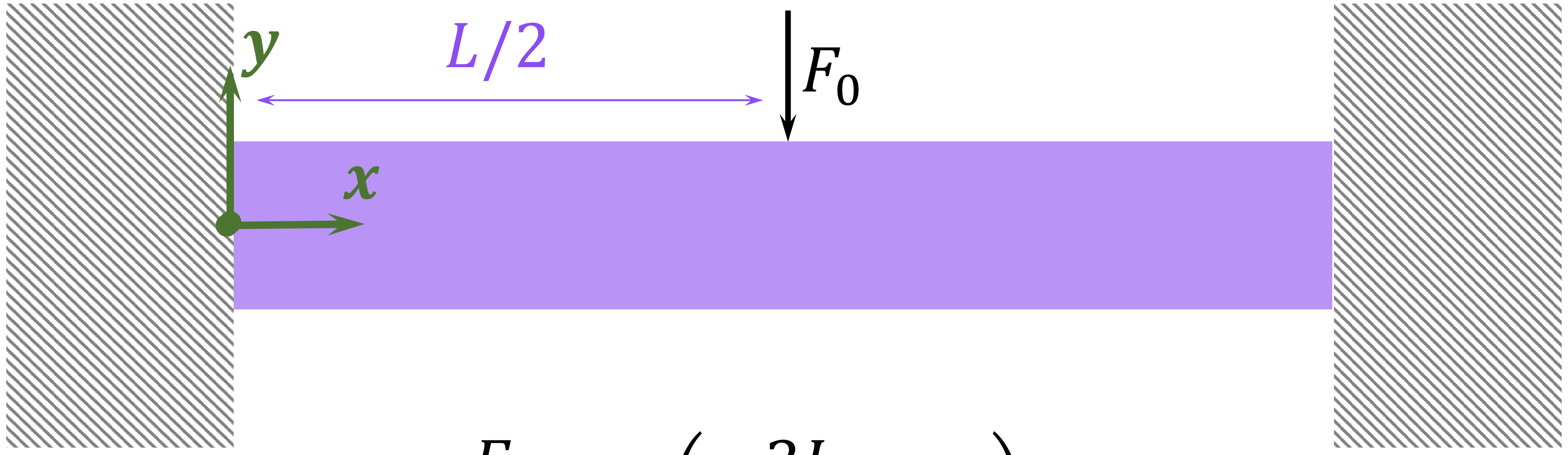
Comparing cantilever and C-C beam





$$w(x) = \frac{F_0}{6EI} x^2 (x - 3L)$$

$$w(L) = -\frac{F_0}{3EI} L^3$$

$$w(x) = \frac{F_0}{24EI} x^2 \left(-\frac{3L}{2} + 2x\right) \quad if\ x \leq L/2$$

$$w\left(\frac{L}{2}\right) = -\frac{F_0}{192EI} L^3$$

Dimensions (m)	Cantilever deflection to $1\ \mu N$ (m)	C-C beam deflection to $1\ \mu N$ (m)
$10 \times 1 \times 0.1$	10^{-11}	10^{-13}
$0.1 \times 10^{-3} \times 10^{-3}$	10^{-8}	10^{-10}
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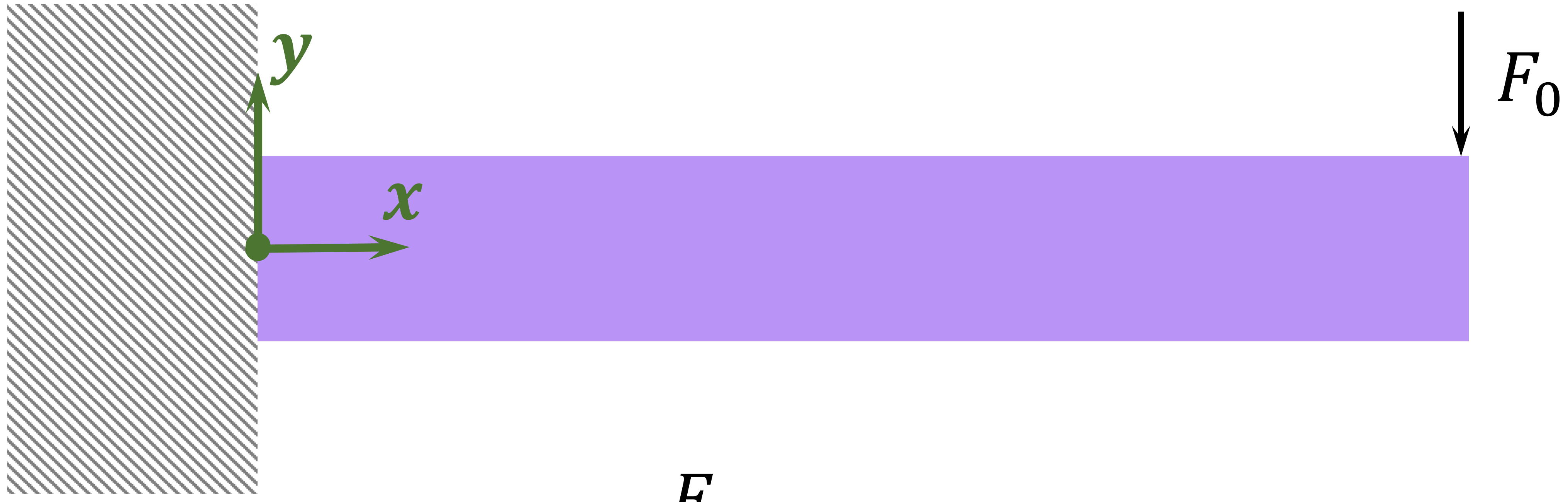
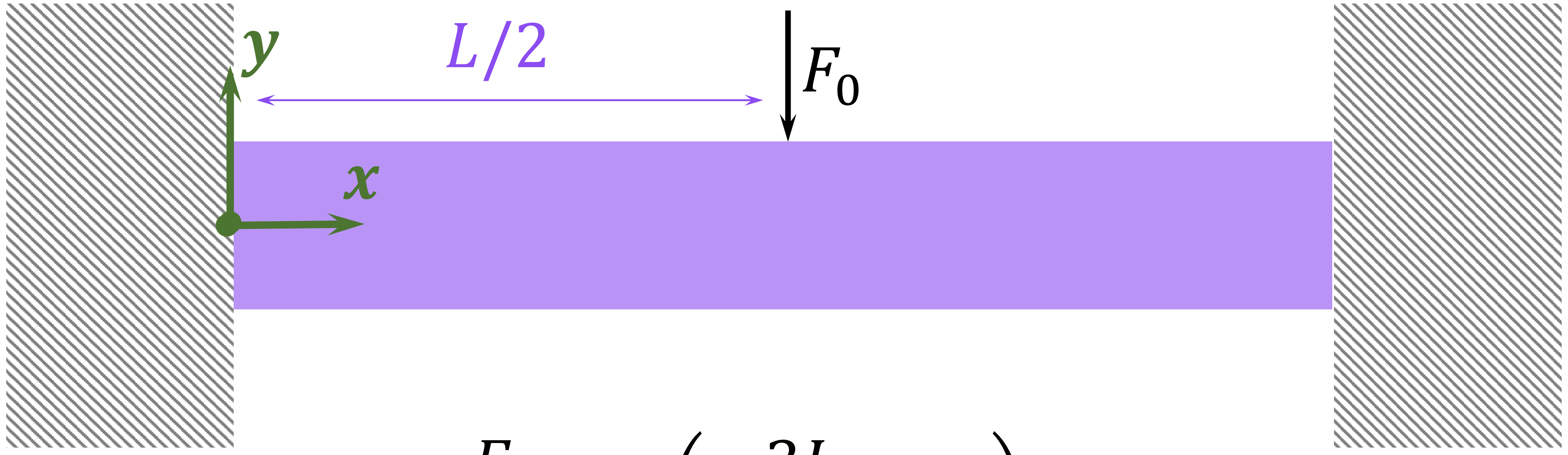
Nonlinearity

Comparing cantilever and C-C beam





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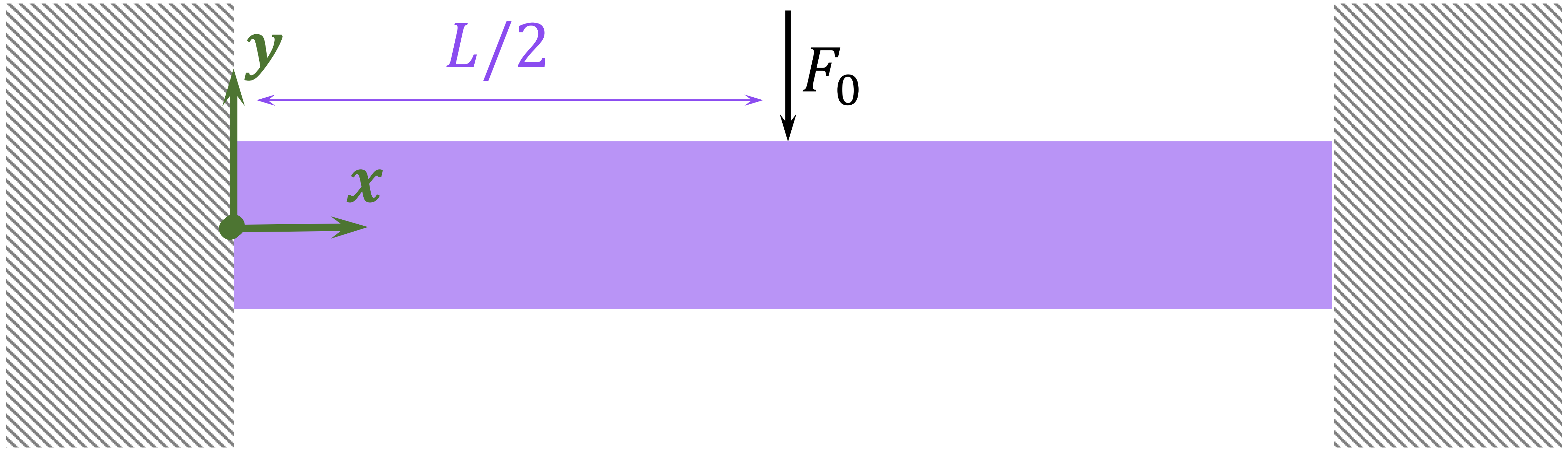
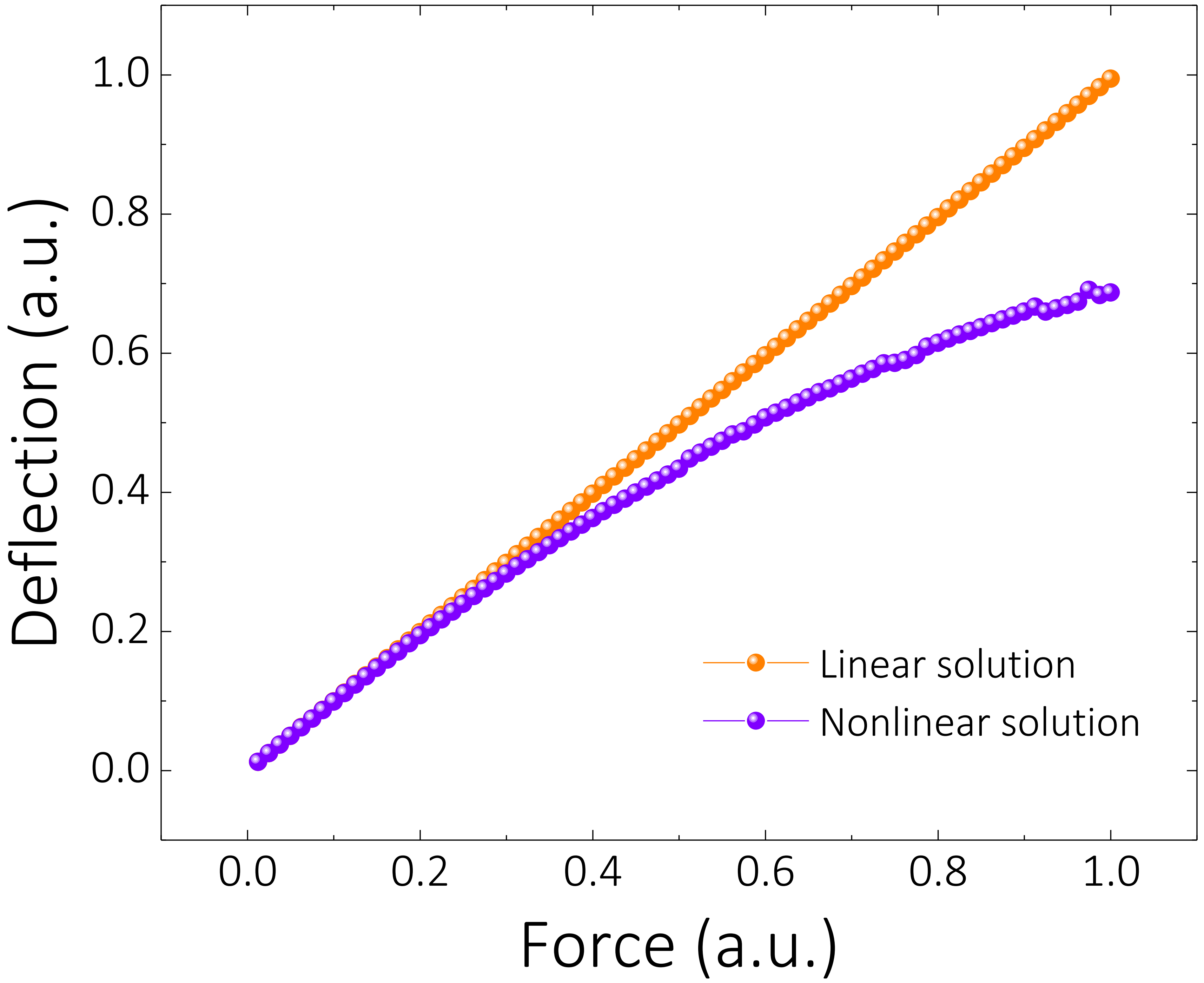
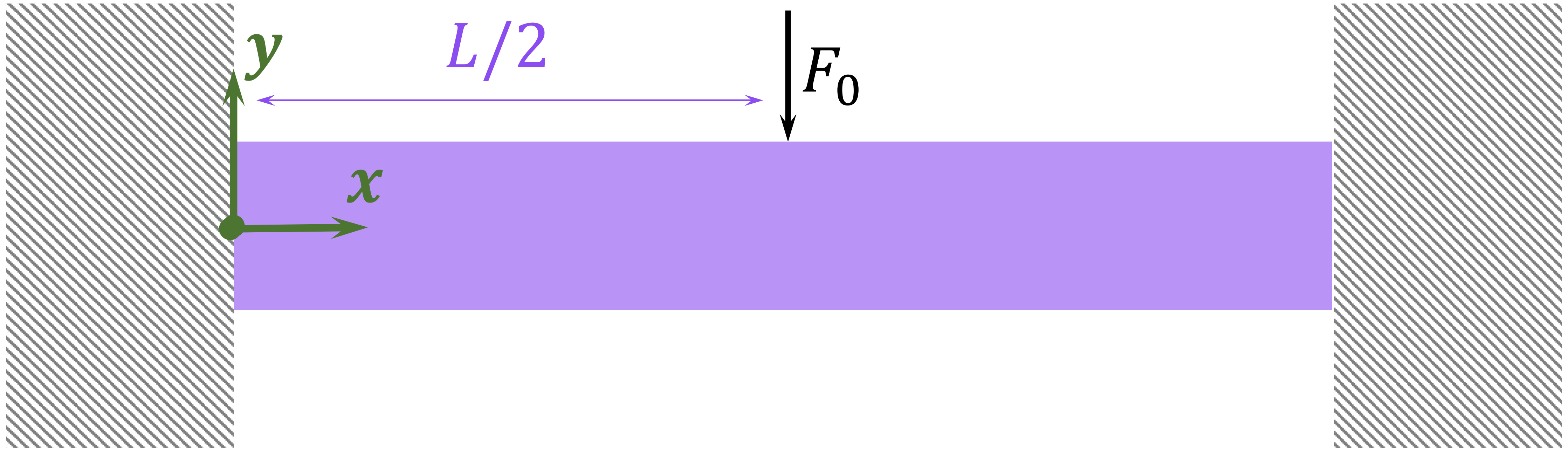
Nonlinear effects





Nonlinear effects

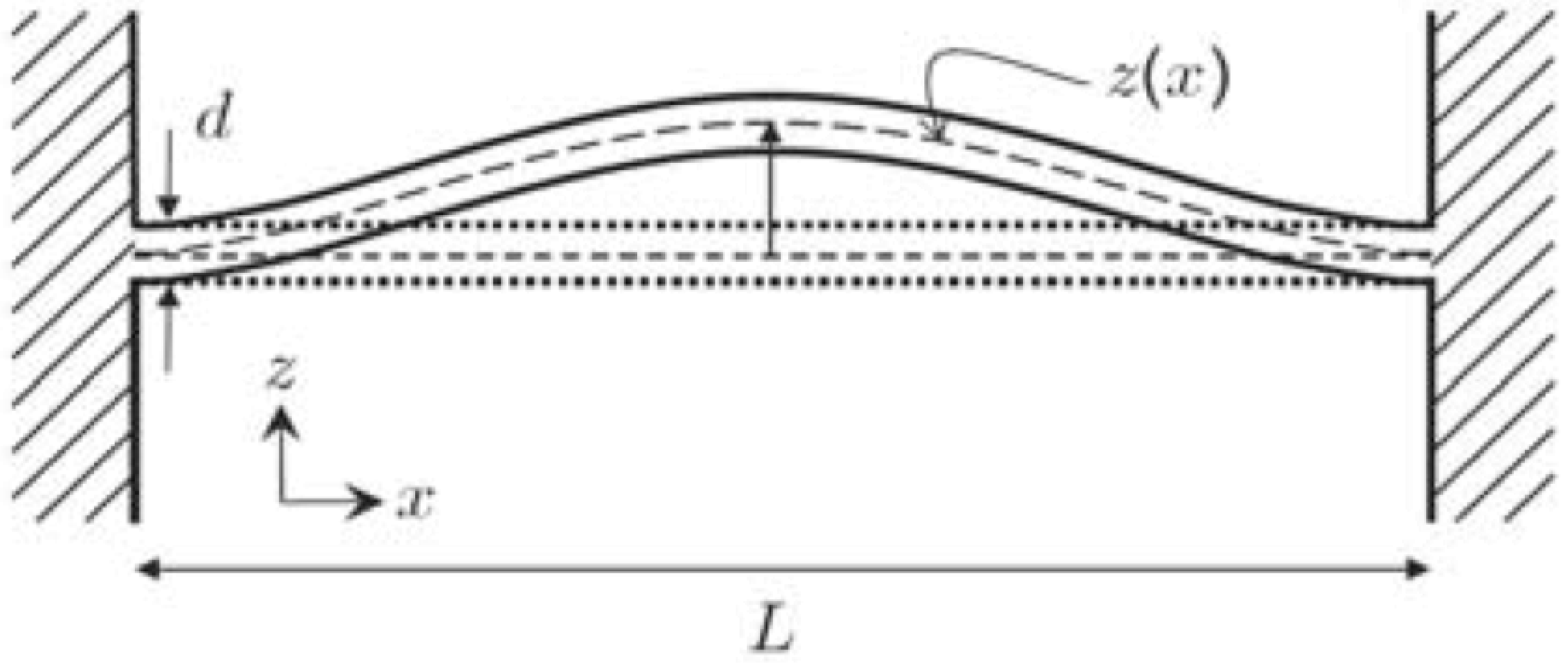


- Nonlinearity happens when the ratio between force and displacement is not constant

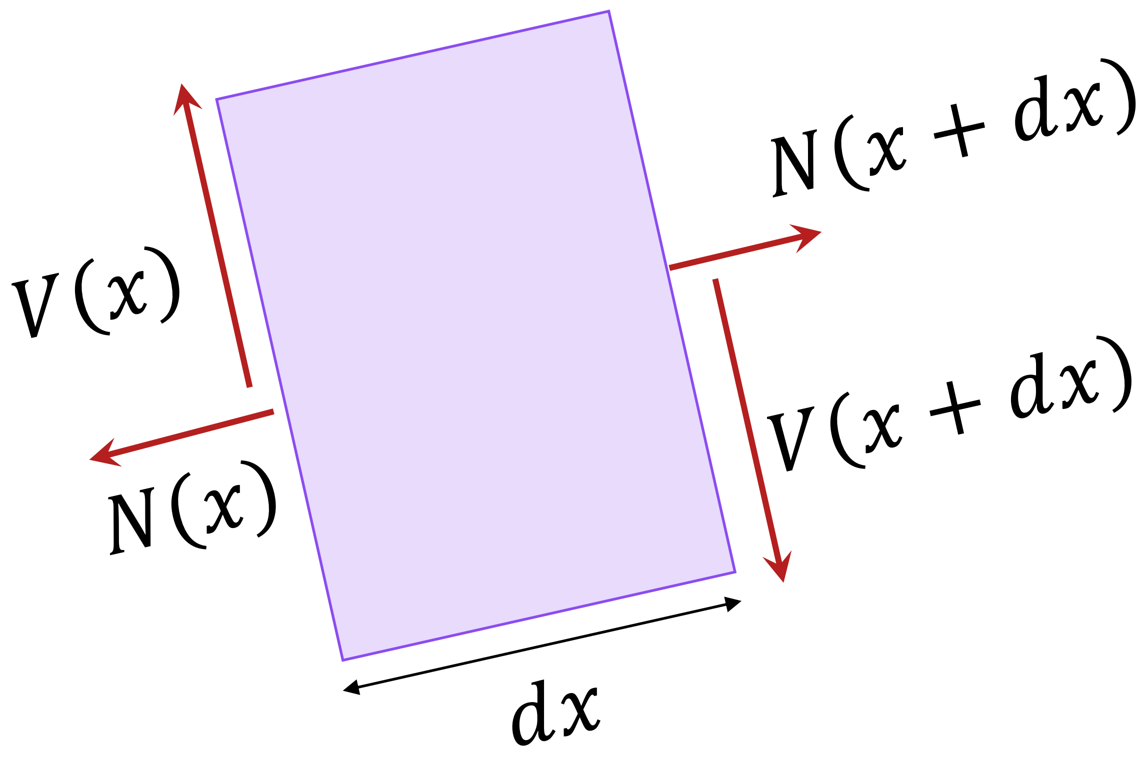
- This is caused by additional terms not considered in the description of the deflection

- Cantilever: Small displacements approximation is not valid $\kappa = \frac{1}{\rho} = \frac{w''(x)}{(1+w'(x)^2)^{3/2}}$
  - Nonlinearity happens when deflection is $w(L) \sim L$
- C-C Beam: Built-in Tension in the beam modifies the stiffness
  - Nonlinearity happens when deflection is $w(L/2) \sim t$

Built-in tension



- Deflection of the beam gives rise to an elongation caused by the motion:
  - $\frac{\Delta L}{L} = \varepsilon_x = \frac{1}{2L}\int_0^L w'(x)^2\, dx$
- This required elongation causes a stress to appear in the beam
  - $\sigma_x = E\varepsilon_x \rightarrow N = E\varepsilon_x A$



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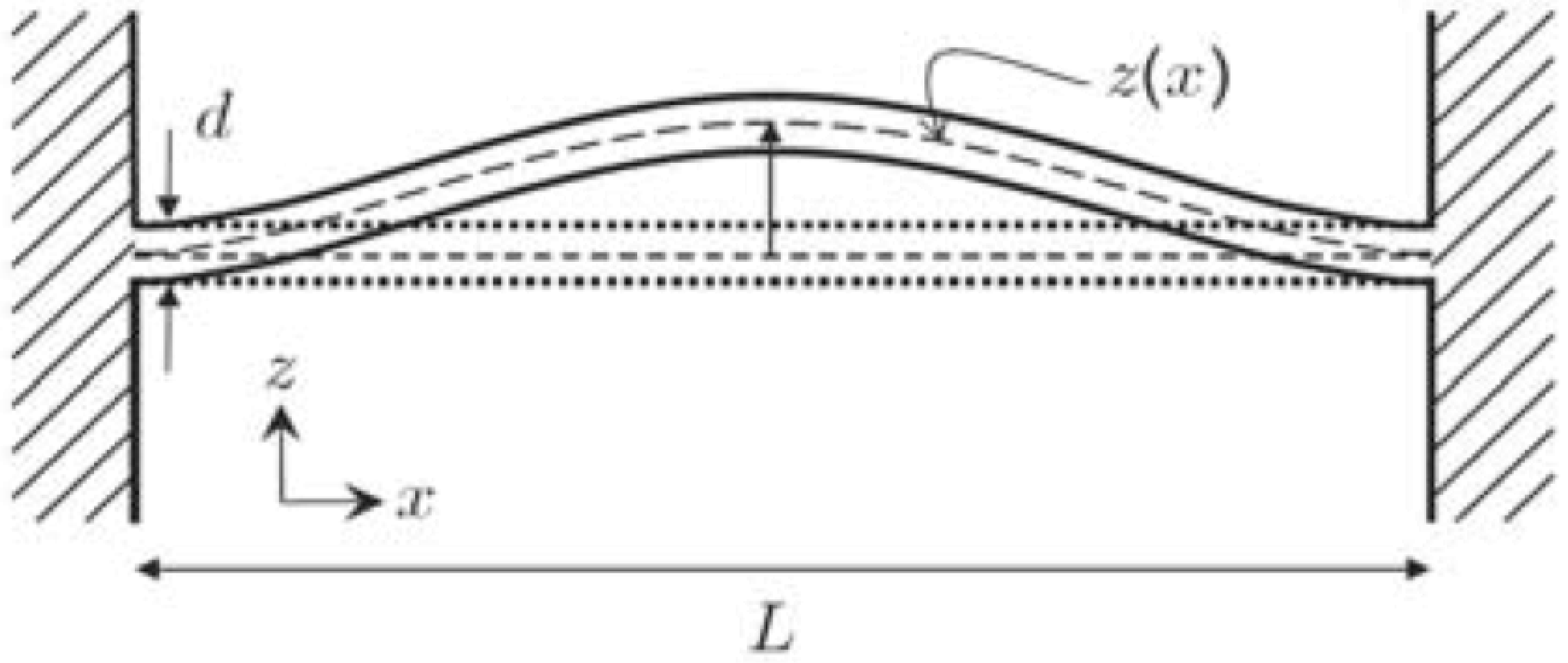

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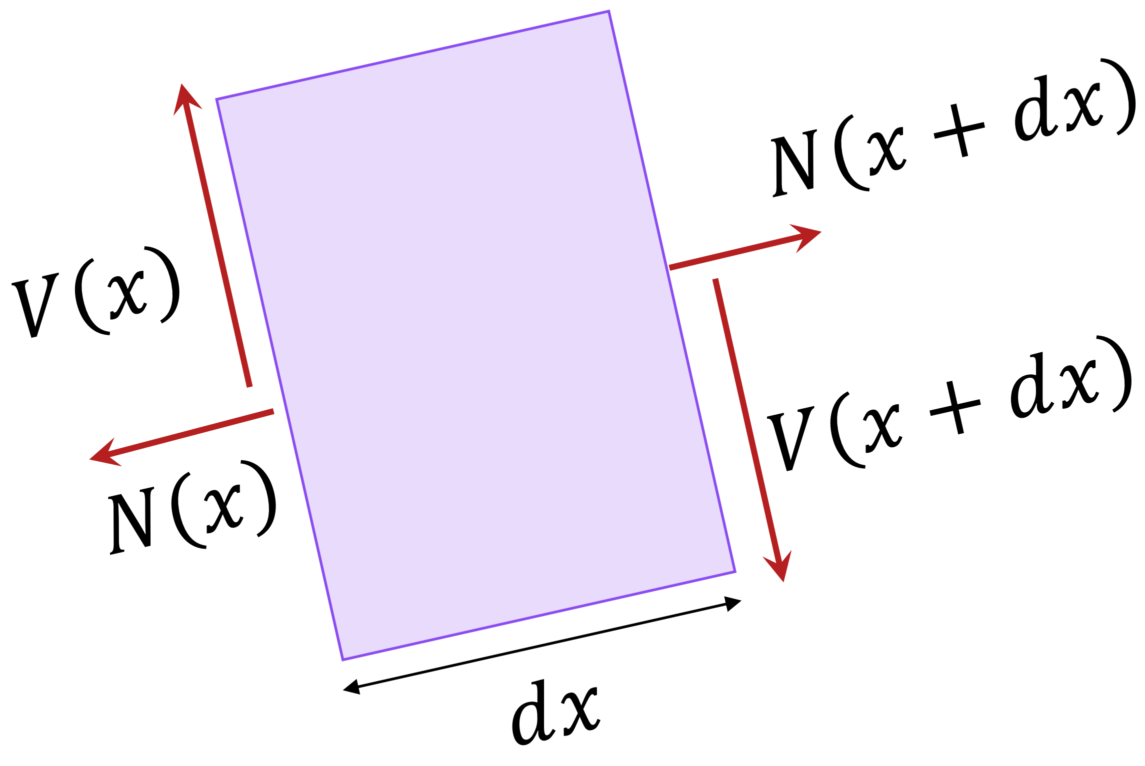
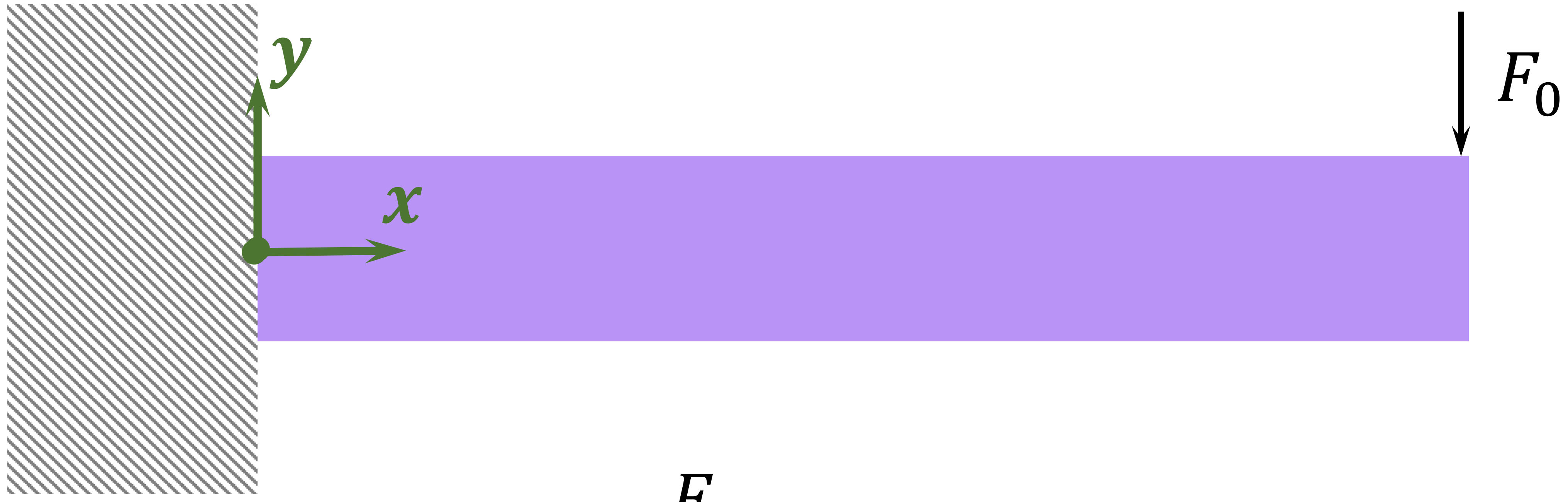
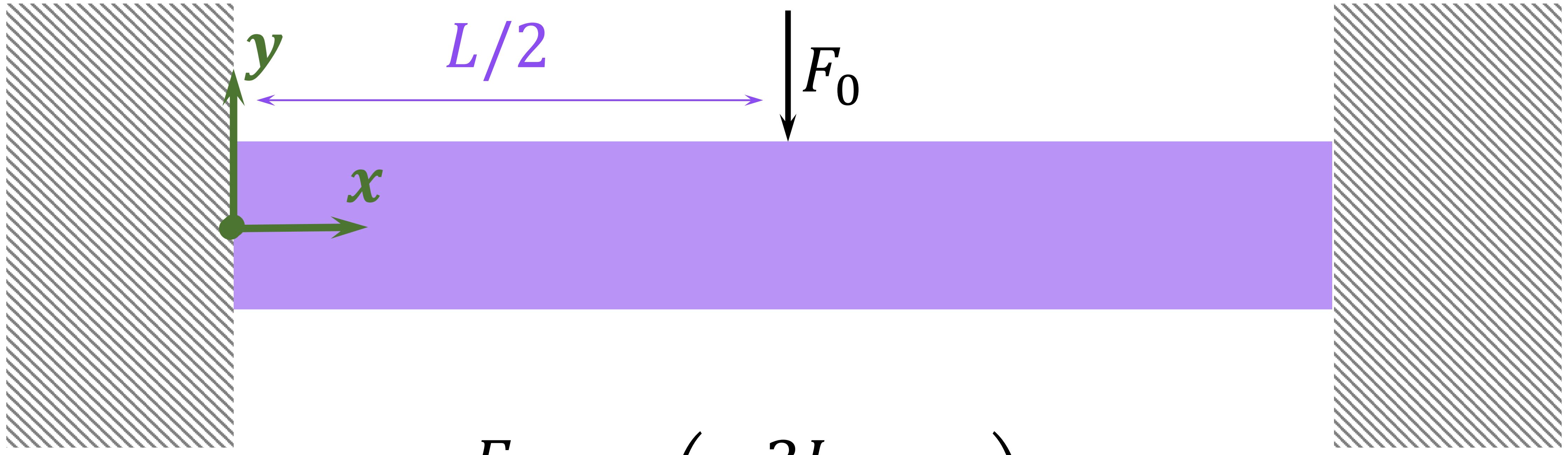
$$-V(x+dx) + V(x) + N\cdot\left(w'(x+dx) - w'(x)\right) =$$
$$= -V'(x)\,dx + Nw''(x)dx = -V'(x)\,dx + Nw''(x)dx =$$
$$= -V'(x)\,dx + \frac{EA}{2L}\int_0^L w'(x)^2\,dx\,w''(x)dx$$



Composite beams

Comparing cantilever and C-C beam





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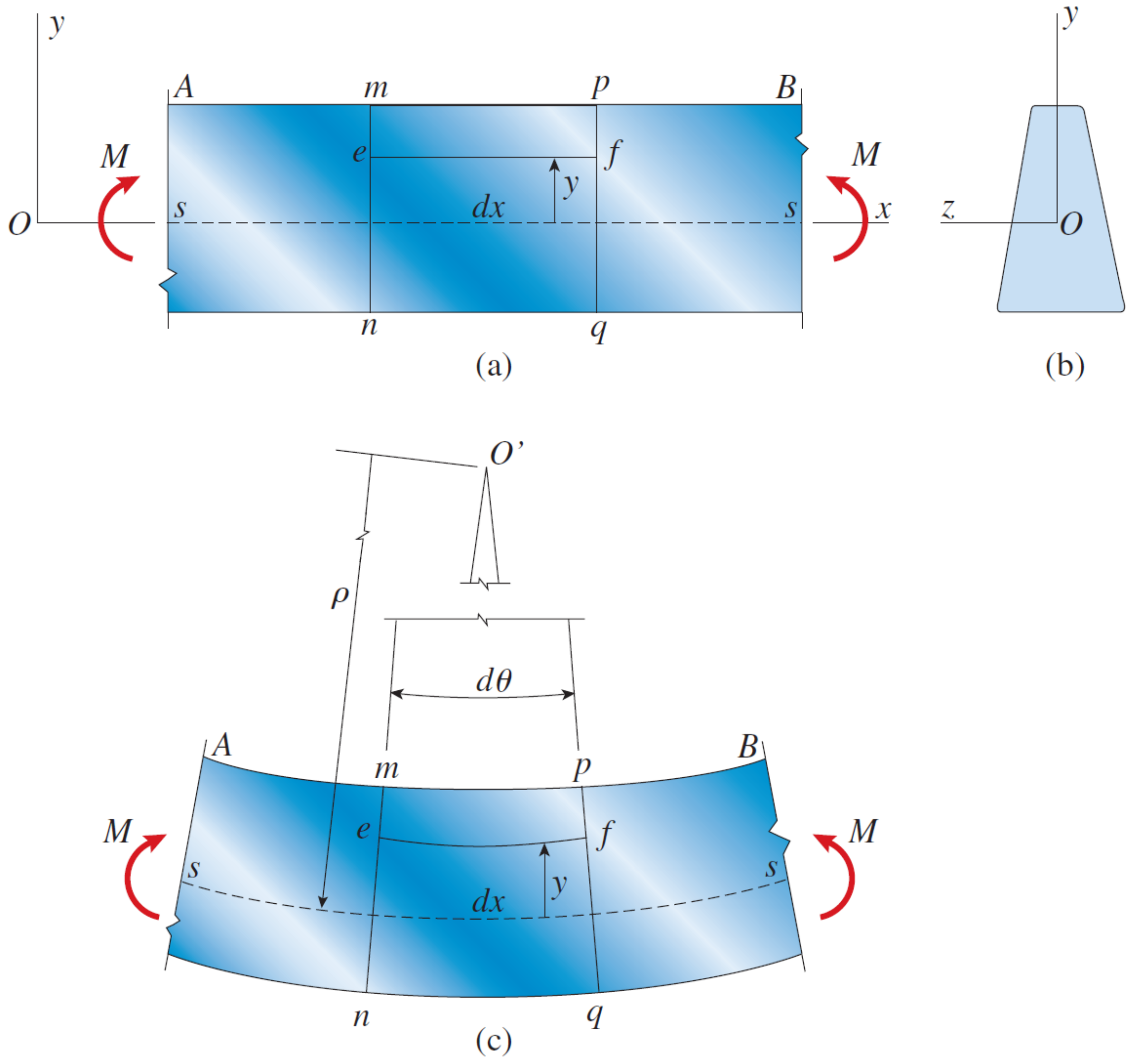
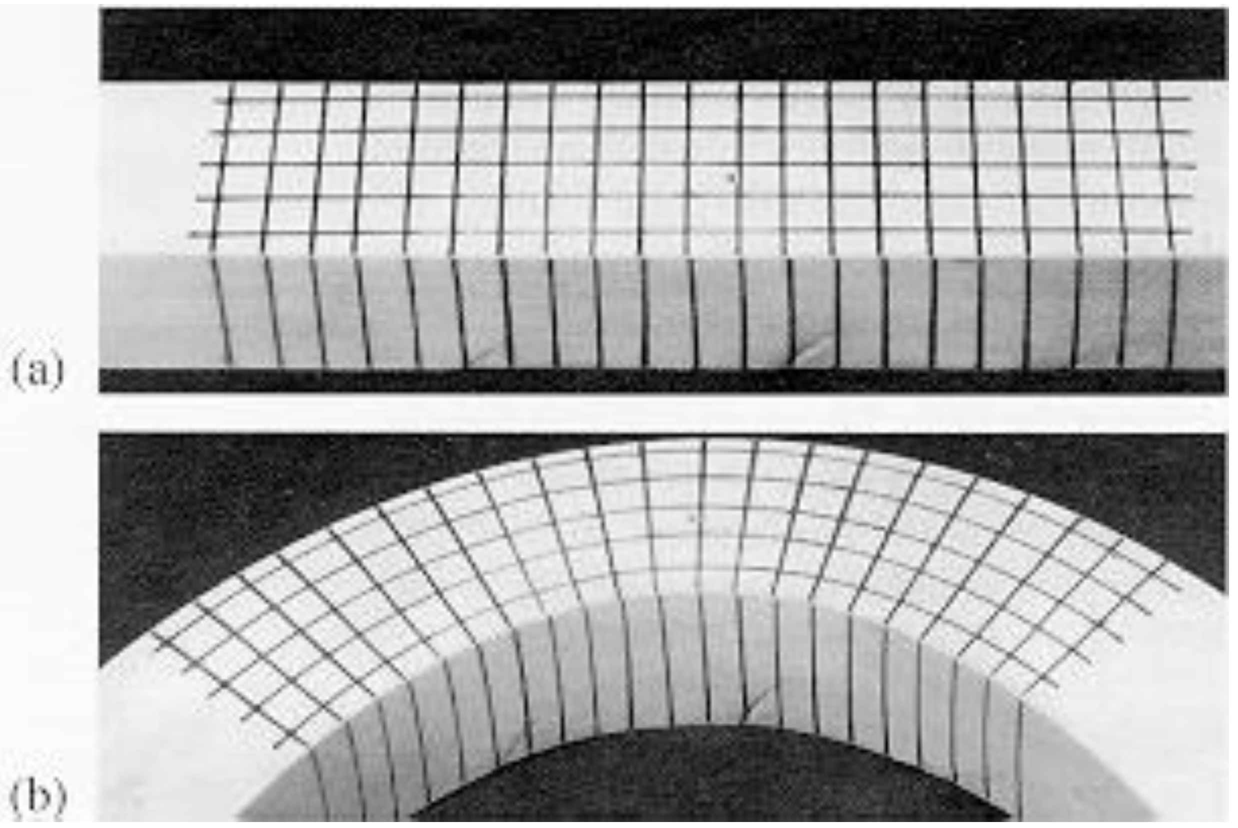
Normal Strain in Composite Beams

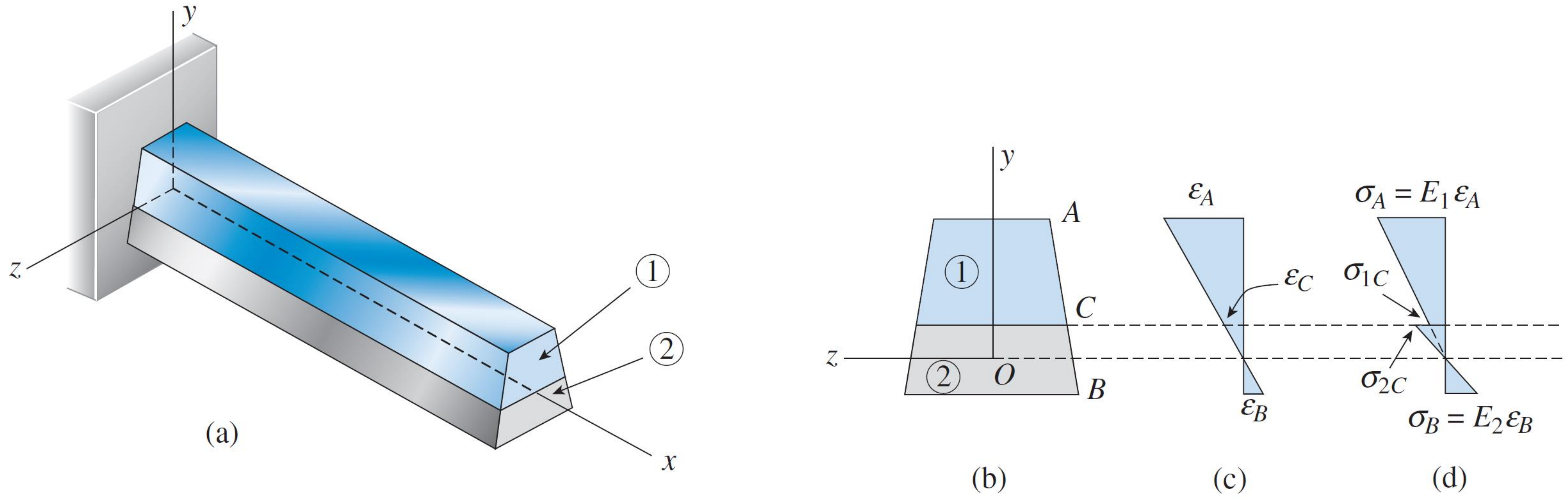
- When a beam is bending, we can locally define a radius of curvature ρ and a curvature κ
- Planes normal to the beam axis before bending will remain plane after bending
- Part of the beam will expand while part of the beam will contract
- The axis where there is no elongation is called Neutral Axis





Normal Strain in Composite Beams

- For the curvature κ of the beam we get: $\kappa = \frac{\mathrm{d}\theta}{\mathrm{d}s} = \frac{1}{\rho}$
- After deformation, the neutral axis remains the same
- Any other parallel line:
- $L_f = \left(\rho - (y - y_0)\right)\mathrm{d}\theta = \mathrm{d}s_0 - (y - y_0)\frac{\mathrm{d}s_0}{\rho} \rightarrow \varepsilon_x = -\frac{y-y_0}{\rho}$



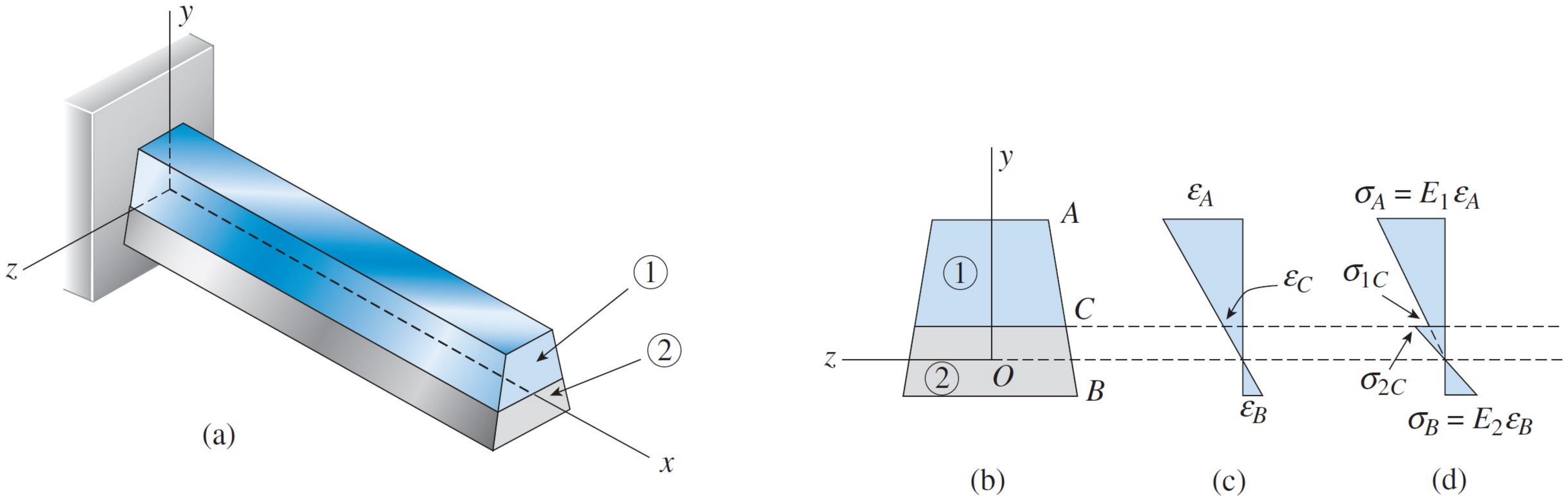
This is the same for a composite or a monomaterial beam

Normal Stress in Composite Beams

- $\varepsilon_x = -\frac{y-y_0}{\rho}$
- What changes is the calculation of stress
- Using Hooke's law: ~~$\sigma_x = E\varepsilon_x = -E\frac{y-y_0}{\rho}$~~

$$\sigma_{x,i} = E_i\varepsilon_x = -E_i\frac{y-y_0}{\rho}$$

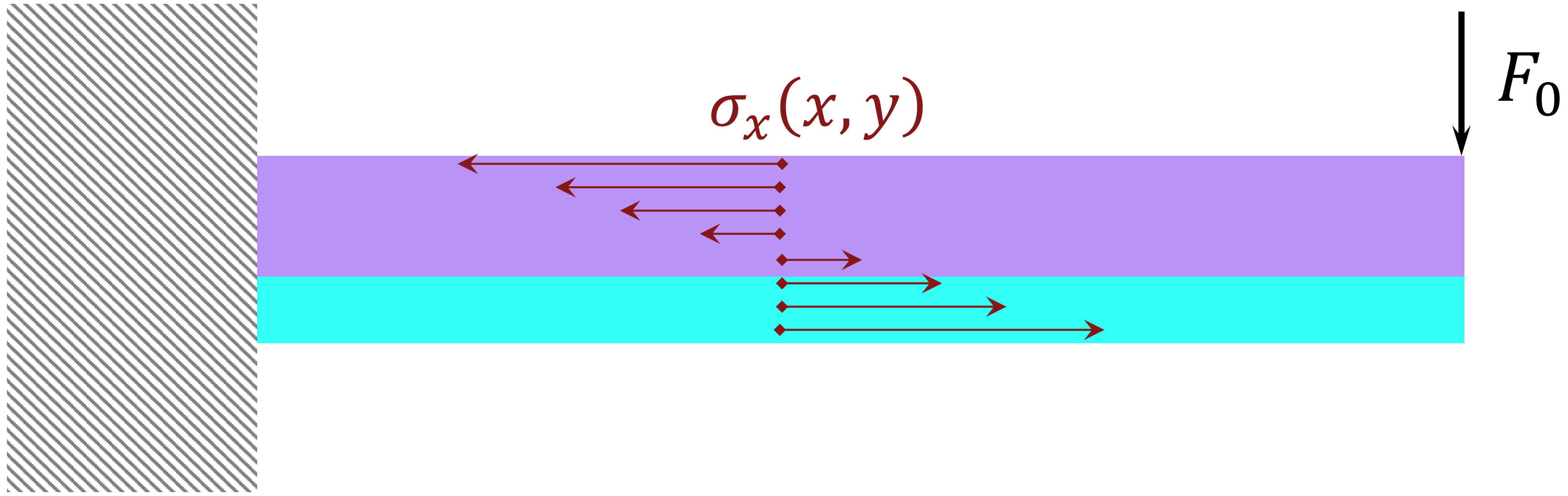
$$\sigma_x(y) = -E(y)\frac{y-y_0}{\rho}$$



Neutral axis

- We "cut" and we apply equilibrium:

$$\sum F_x = N = 0$$



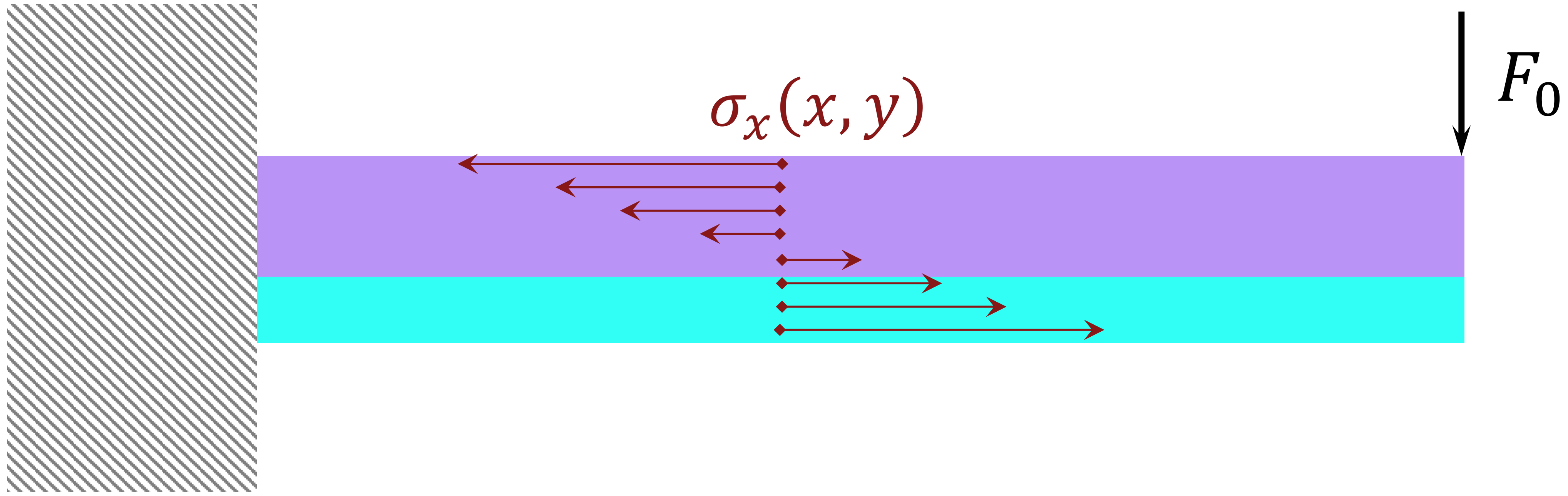
Neutral axis

- We "cut" and we apply equilibrium:

$$\sum F_x = N = 0$$

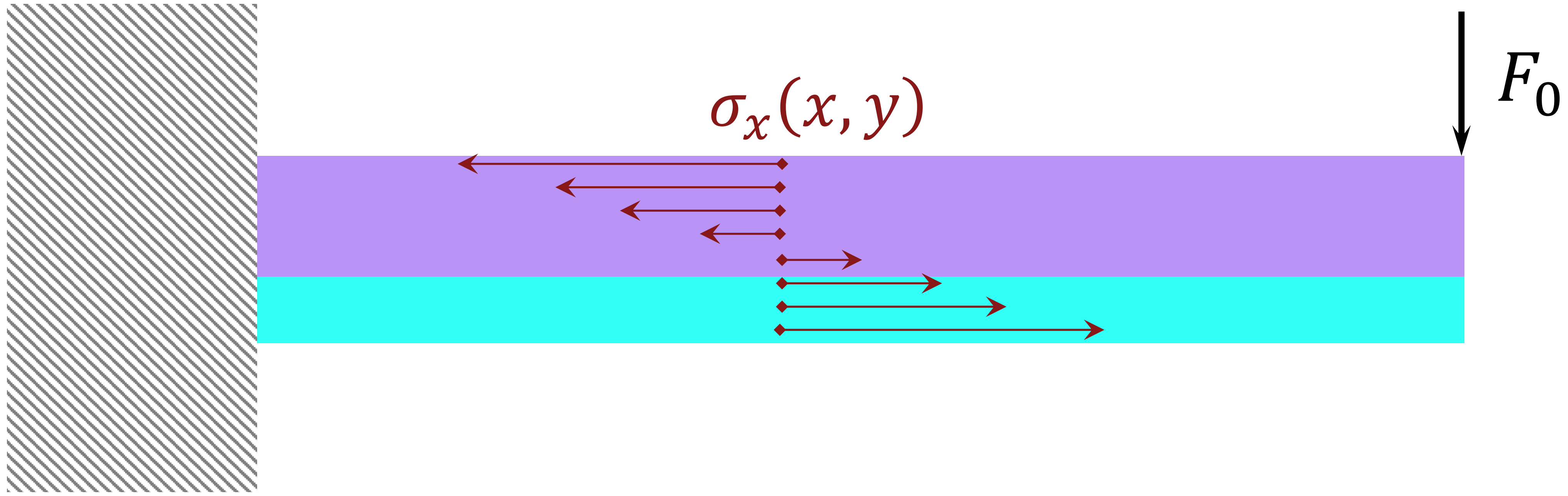
$$N = \iint \sigma_x(x,y)\,dy\,dz = \sum_i \iint E_i \varepsilon_x(x,z)\,dy\,dz = -\sum_i \iint E_i \frac{(y-y_0)}{\rho}\,dy\,dz$$

$$y_0 = \frac{\sum_i \int E_i \frac{y}{\rho}\,dA}{\sum_i \int \frac{E_i}{\rho}\,dA} = \frac{\sum_i \int E_i y\,dA}{\sum_i \int E_i\,dA} = \frac{\sum_i E_i \int y\,dA}{\sum_i E_i \int dA} \rightarrow \textbf{\textit{not the centroid}}$$



Bending Moment

- We can calculate the moment created by the normal stresses with respect to the neutral axis



Bending Moment

- We can calculate the moment created by the normal stresses with respect to the neutral axis

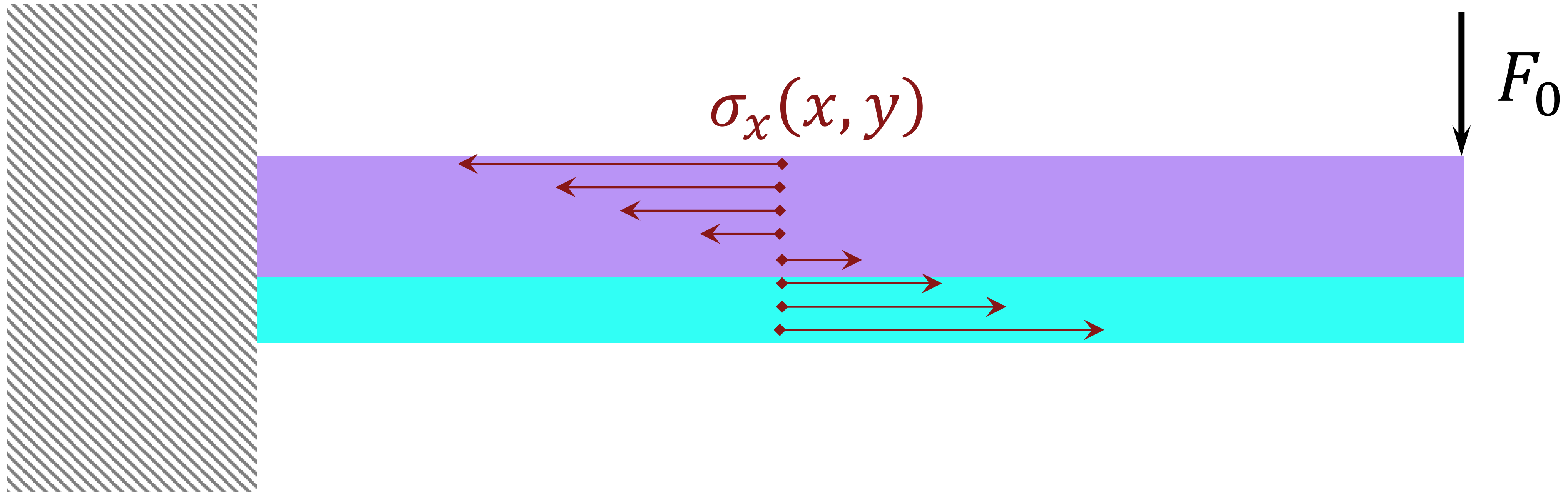
$$M_z(x) = -\iint \sigma_x(x,y)(y-y_0)\,\mathrm{d}A = \sum_i \iint E_i \frac{(y-y_0)^2}{\rho}\,\mathrm{d}A$$

Bending Stiffness

$$M_z(x) = \frac{1}{\rho}\langle EI_{z,y_0}\rangle;\ \text{where}\ \langle EI_{z,y_0}\rangle = \sum_i E_i \iint (y-y_0)^2\,\mathrm{d}A = \sum_i E_i I_{z,y_0,i}$$

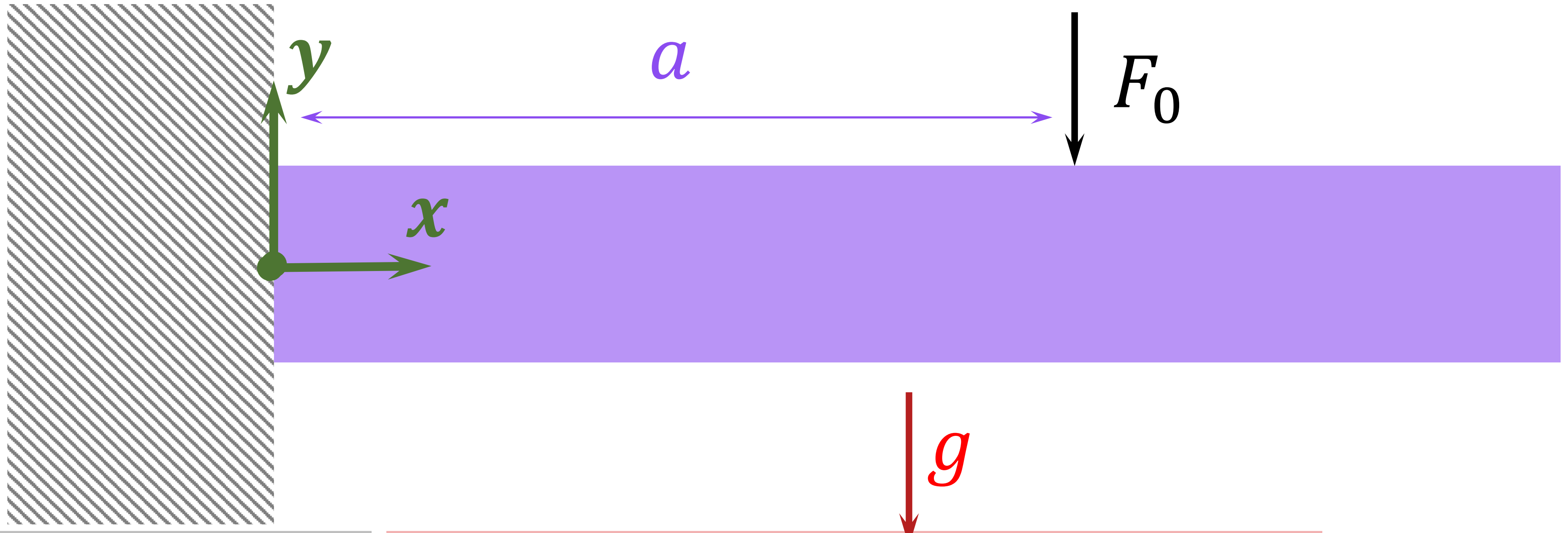
$$\sigma_x(x,y) = -\frac{E(y)M_z(x)}{\langle EI_{z,y_0}\rangle}(y-y_0)$$

Flexure formula for composite beams



Accelerometers

Let's put some numbers

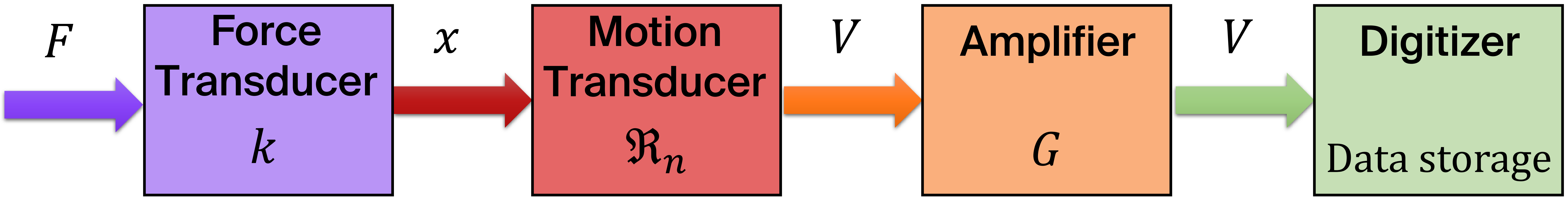


$$w(x) = \begin{cases} \frac{F_0}{6EI}x^2(x-3a) - \frac{\rho g A}{12EI}x^2(6L^2-4Lx+x^2) & if\ x \le a \\ \frac{F_0}{6EI}a(-2a^2-3xa) - \frac{\rho g A}{12EI}x^2(6L^2-4Lx+x^2) & if\ x \ge a \end{cases}$$

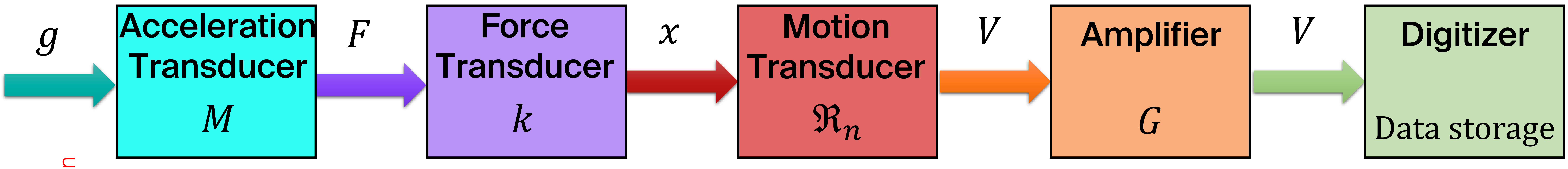
Device	Dimensions (m)	Max. deflection gravity (m)	Force to match gravity (N)
Diving board	$10 \times 1 \times 0.1$	0.3	$25 \cdot 10^3$
Tuning fork	$0.1 \times 10^{-3} \times 10^{-3}$	$3 \cdot 10^{-5}$	$2 \cdot 10^{-3}$
Microcantilever	$(500 \times 50 \times 5) \cdot 10^{-6}$	10^{-9}	$4 \cdot 10^{-9}$
Nanocantilever	$(10 \times 0.5 \times 0.1) \cdot 10^{-6}$	$3 \cdot 10^{-13}$	10^{-14}

In micro/nano scale, gravity and weight can be neglected – but maybe important for vibrations

Force detection



Acceleration detection



Design example

- Large mass
- Soft spring (supporting beam)
- Small gaps